Cathodyne or Concertina Phase Splitter (CPS) Derivations

Fix-biased version

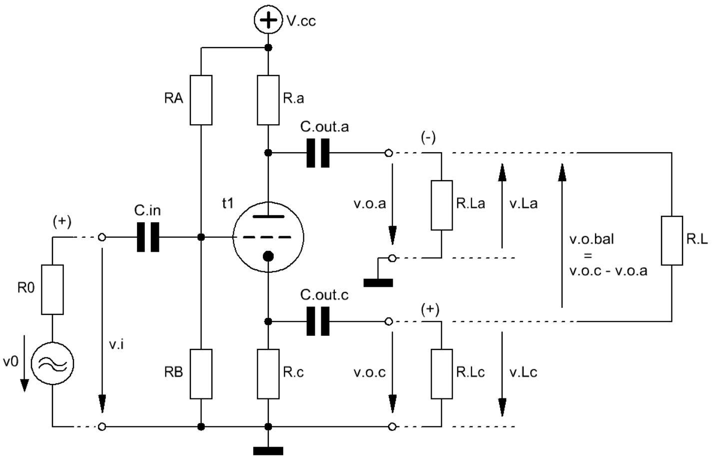

Fig. 10.1a Basic CPS circuit with fixed bias

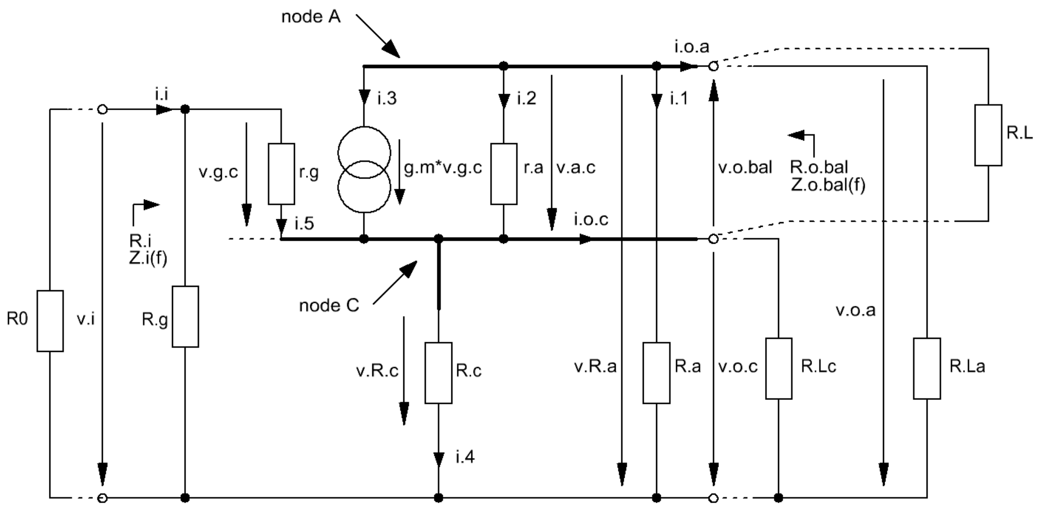

Fig. 10.7a Equivalent circuit of Fig. 10.1a

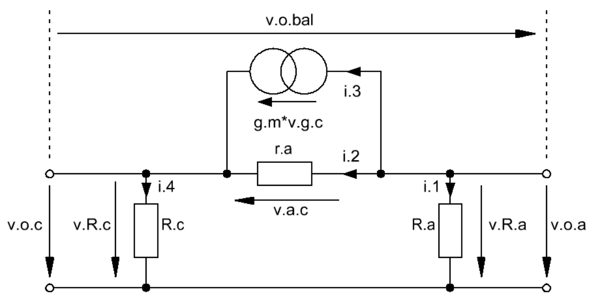

Fig. 10.8a Stripped down equivalent circuit

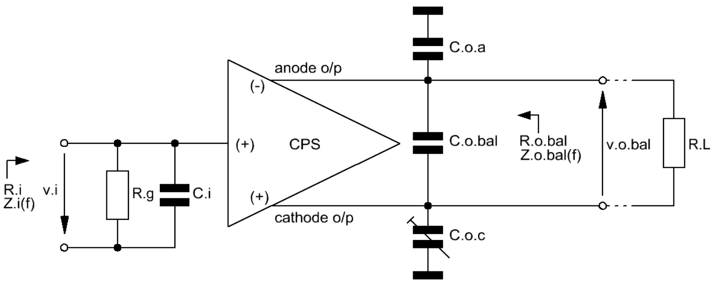

Fig. 10.9 Un-balanced to balanced conversion

Circuit variables à la S. Y.

$g_m := 7.2\cdot 10^{-3} S$ $\quad \mu := 31$ $\quad r_a := \frac{\mu}{g_m}$ $\quad r_a = 4.306\times 10^3\ \Omega$

$R_{c1} := 1.4\cdot 10^3\Omega$ $\quad R_c := 15\cdot 10^3\Omega$ $\quad R_a := 15\cdot 10^3\Omega$ $\quad V_g := -1.73V$

$I_a := 5.4\cdot 10^{-3}A$ $\quad V_a := 88V$ $\quad V_{cc} := V_a + I_a\cdot(R_a + R_c)$ $\quad V_{cc} = 250\ V$

$RA := 2.21\cdot 10^6\Omega$ $\quad RB := RA\cdot\left(\frac{V_{cc} - I_a\cdot R_c - V_a + |V_g|}{V_{cc} - I_a\cdot R_c - |V_g|}\right)$ $\quad RB = 1.093\times 10^6\ \Omega$

$R_g := \left(\frac{1}{RA} + \frac{1}{RB}\right)^{-1}$ $\quad R_g = 731.333\times 10^3\ \Omega$ $\quad V_{cc} - I_a\cdot R_c - V_a + |V_g| = 82.73\ V$

$V_{cc} - I_a\cdot R_c - |V_g| = 167.27\ V$

1. CPS used as un-balanced to balanced converter

1.1 Gains

1.1.1 Idle gains G_0

cathode gain G_c

$$G_{0.c} := \mu\cdot\frac{R_c}{r_a + R_a + (1+\mu)\cdot R_c} \qquad G_{0.c} = 931.293\times 10^{-3}$$

anode gain G_a

$$G_{0.a} := -\mu\cdot\frac{R_a}{r_a + R_a + (1+\mu)\cdot R_c} \qquad G_{0.a} = -931.293\times 10^{-3}$$

equal loads R lead to the general idle gain equation G_0 and $G_{0.bal}$

$$R_a = R_c = R \quad => \quad |G_{0.a}| = G_{0.c} = G_0 = \mu\cdot\frac{R}{r_a + (2+\mu)\cdot R}$$

$v_{o.bal} = v_{o.c} - v_{o.a}$ $\quad v_{o.c} = G_{0.c}\cdot v_i$ $\quad v_{o.a} = G_{0.a}\cdot v_i$

=> $v_{o.bal} = G_{0.c} \cdot v_i - G_{0.a} \cdot v_i = v_i \cdot 2 \cdot G_0$ => $G_{0.bal} = 2 \cdot G_0 = \frac{2 \cdot \mu \cdot R}{r_a + (2+\mu) \cdot R}$

Idle balanced gain $G_{0.bal}$

$R := R_a$ => $G_{0.bal} := \frac{2 \cdot \mu \cdot R}{r_a + (2+\mu) \cdot R}$ $G_{0.bal} = 1.863 \times 10^0$

1.1.2 Gains $G_{bal}(R_L)$ with o/p loaded and balanced o/p resistance $R_{o.bal}$

balanced o/p load = R_L, hence, the additional loads to the existing R_a and R_c is the paralleling of $0.5 * R_L$ each

$R_L := 10 \cdot 10^3 \Omega$ => $R_{L.a} := \frac{R_L}{2}$ $R_{L.c} := R_{L.a}$

Output load dependent gain:
any decrease or increase of the anode and cathode resistors from R to eg. $R_{new} = R \| (0.5 * R_L)$ automatically leads to a changed $G_{0.bal}$ via the new $G_{0.a.new} = G_{0.c.new} = G_{0.new}$, thus changing the balanced output resistance too.

$R_{new}(R_L) := \left(\frac{1}{R} + \frac{1}{0.5 \cdot R_L} \right)^{-1}$ $R_{new}(R_L) = 3.75 \times 10^3 \ \Omega$

=> $G_{bal1}(R_L) := \frac{2 \cdot \mu \cdot \left(\frac{1}{R} + \frac{1}{0.5 \cdot R_L} \right)^{-1}}{r_a + (2+\mu) \cdot \left(\frac{1}{R} + \frac{1}{0.5 \cdot R_L} \right)^{-1}}$ $G_{bal1}(R_L) = 1.816 \times 10^0$

=> $\lim_{R_L \to \infty} G_{bal1}(R_L) = G_{0.bal}$ q.e.d see next line

$\lim_{R_L \to \infty} G_{bal1}(R_L) \to .77500000000000000000e21 \cdot \Omega \cdot \frac{S}{3587962962962962963. + .41250000000000000000e21 \cdot \Omega \cdot S} = 1.863$

Simplification of the $G_{bal}(R_L)$ equation

=> $G_{bal}(R_L) := \frac{2 \cdot \mu}{2 + \mu + r_a \cdot R_{new}(R_L)^{-1}}$ $G_{bal}(R_L) = 1.816 \times 10^0$

1.2 Output resistance $R_{o.bal}$ of balanced output

Equivalent circuit of the balanced amp situation:

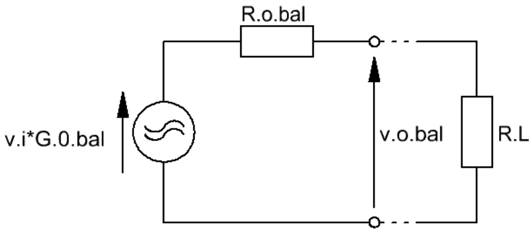

Fig. 10.10 Equivalent circuit of the balanced situation

$$v_{o.bal} = v_i \cdot G_{0.bal} \cdot \frac{R_L}{R_{o.bal} + R_L} \qquad \frac{v_{o.bal}}{v_i} = G_{bal}(R_L) \qquad \Rightarrow \qquad G_{bal}(R_L) = G_{0.bal} \cdot \frac{R_L}{R_{o.bal} + R_L}$$

$$\Rightarrow \qquad R_{o.bal1} := \frac{G_{0.bal} \cdot R_L - G_{bal}(R_L) \cdot R_L}{G_{bal}(R_L)} \qquad R_{o.bal1} = 258.693 \times 10^0 \ \Omega$$

$$R_{o.bal2} := R_L \cdot \left(\frac{G_{0.bal}}{G_{bal}(R_L)} - 1 \right) \qquad R_{o.bal2} = 258.693 \times 10^0 \ \Omega$$

Simplification of the $R_{o.bal}$ equation

$$\left[\frac{\dfrac{2 \cdot \mu \cdot R}{r_a + (2+\mu) \cdot R}}{\dfrac{2 \cdot \mu \cdot \left(\dfrac{1}{R} + \dfrac{1}{0.5 \cdot R_L} \right)^{-1}}{r_a + (2+\mu) \cdot \left(\dfrac{1}{R} + \dfrac{1}{0.5 \cdot R_L} \right)^{-1}}} \cdot R_L - R_L \right] = 258.693 \times 10^0 \ \Omega$$

$$\Rightarrow \qquad R_{o.bal} := \frac{2 \cdot r_a \cdot R}{r_a + (\mu + 2) \cdot R} \qquad R_{o.bal} = 258.693 \times 10^0 \ \Omega$$

1.3 Input and Output Capacitances

C_a and $C_{o.bal}$ plus trimming of $C_{o.c}$ will lead to frequency dependent $G_{bal}(f)$ and o/p impedance $Z_{o.bal}(f)$ as follows:

$f := 10Hz, 20Hz .. 10^5 Hz$

$C_{g.c} := 3.1 \cdot 10^{-12} F \qquad C_{g.a} := 1.4 \cdot 10^{-12} F \qquad C_{a.c} := 3.0 \cdot 10^{-12} F$

$C_{o.bal} := C_{a.c}$

$$Z_{o.bal}(f) := \left(\frac{1}{R_{o.bal}} + 2j\cdot\pi\cdot f\cdot C_{o.bal}\right)^{-1}$$

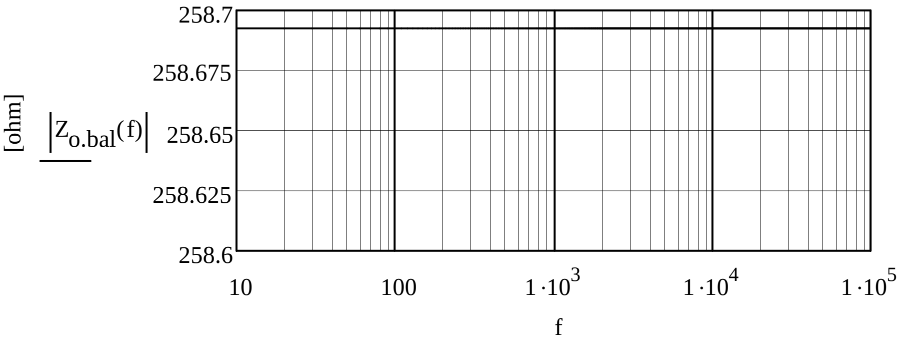

Fig. 10.11 Output impedance vs. frequency

$$G_a(R_L) := \frac{-\mu\cdot\left(\frac{1}{R} + \frac{1}{0.5\cdot R_L}\right)^{-1}}{r_a + (2+\mu)\cdot\left(\frac{1}{R} + \frac{1}{0.5\cdot R_L}\right)^{-1}} \qquad G_c(R_L) := \frac{\mu\cdot\left(\frac{1}{R} + \frac{1}{0.5\cdot R_L}\right)^{-1}}{r_a + (2+\mu)\cdot\left(\frac{1}{R} + \frac{1}{0.5\cdot R_L}\right)^{-1}}$$

$$C_i := C_{g.a}\cdot\left(1 - G_a(R_L)\right) + C_{g.c}\cdot\left(1 - G_c(R_L)\right) \qquad C_i = 2.957\times 10^{-12}\ \mathrm{F}$$

$$Z_i(f) := \left(\frac{1}{R_g} + 2j\cdot\pi\cdot f\cdot C_i\right)^{-1}$$

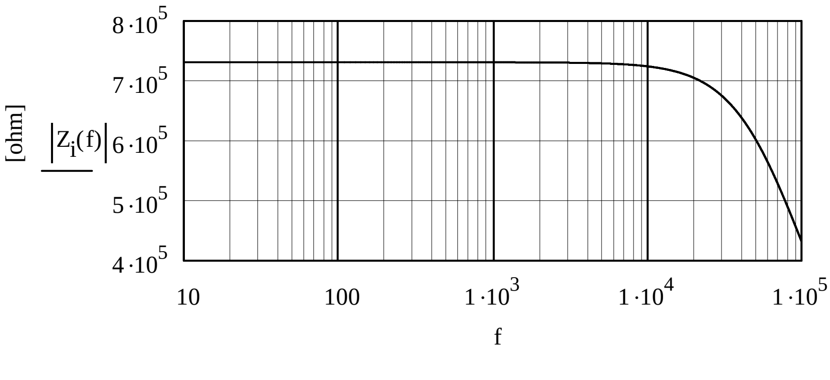

Fig. 10.12 Input impedance vs. frequency

$R_L := 10^3\Omega, 1.5\cdot 10^3\Omega .. 100\cdot 10^3\Omega$

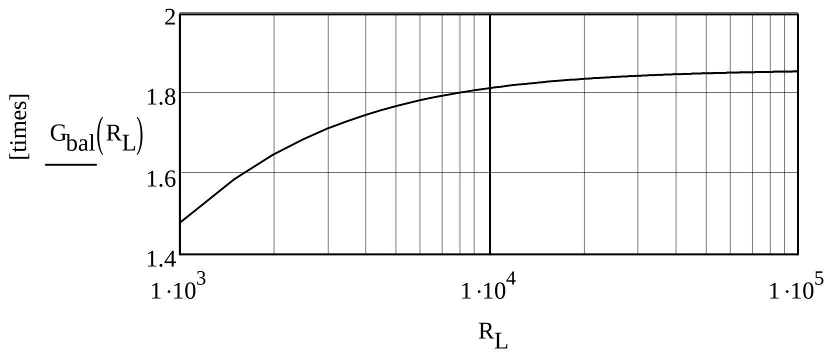

Fig. 10.13 Gain of the un-balanced to balanced converter PCS vs. balanced output load

$$R_{o.bal}(R_L) := R_L \cdot \left(\frac{G_{0.bal}}{G_{bal}(R_L)} - 1 \right)$$

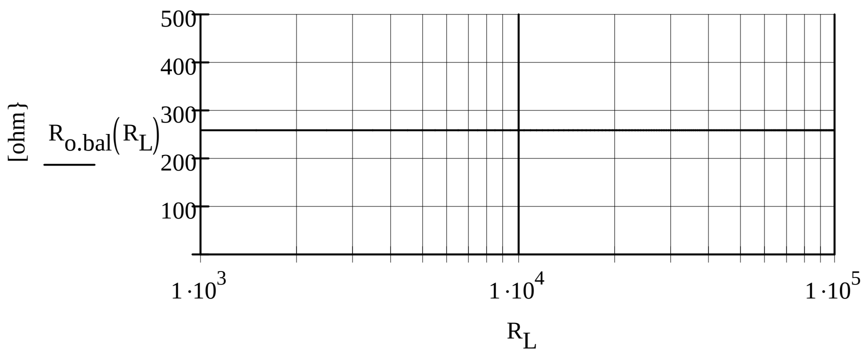

Fig. 10.14 Output resistance of the balanced output vs. balanced output load

1.4 Equal capacitances parallel to R = R_a = R_c

$C = C_a = C_c$ => $Z_a(f) = Z_c(f) = Z(f)$ $\qquad R_L := 7.8 \cdot 10^6 \Omega$

$C_1 := 1 \cdot 10^{-9} F \qquad C_2 := 10 \cdot 10^{-9} F \qquad C_3 := 33 \cdot 10^{-9} F \qquad C_4 := 100 \cdot 10^{-9} F$

$$Z_1(f) := \left(\frac{1}{R} + 2j \cdot \pi \cdot f \cdot C_1 \right)^{-1} \qquad Z_2(f) := \left(\frac{1}{R} + 2j \cdot \pi \cdot f \cdot C_2 \right)^{-1} \qquad Z_3(f) := \left(\frac{1}{R} + 2j \cdot \pi \cdot f \cdot C_3 \right)^{-1} \qquad Z_4(f) := \left(\frac{1}{R} + 2j \cdot \pi \cdot f \cdot C_4 \right)^{-1}$$

$$\Rightarrow \quad G_{bal1}(f, R_L) := \frac{2 \cdot \mu}{2 + \mu + r_a \cdot \left(\frac{1}{Z_1(f)} + \frac{1}{0.5 \cdot R_L} \right)} \qquad G_{bal2}(f, R_L) := \frac{2 \cdot \mu}{2 + \mu + r_a \cdot \left(\frac{1}{Z_2(f)} + \frac{1}{0.5 \cdot R_L} \right)}$$

$$G_{bal3}(f,R_L) := \frac{2\cdot\mu}{2+\mu+r_a\cdot\left(\frac{1}{Z_3(f)}+\frac{1}{0.5\cdot R_L}\right)} \qquad G_{bal4}(f,R_L) := \frac{2\cdot\mu}{2+\mu+r_a\cdot\left(\frac{1}{Z_4(f)}+\frac{1}{0.5\cdot R_L}\right)}$$

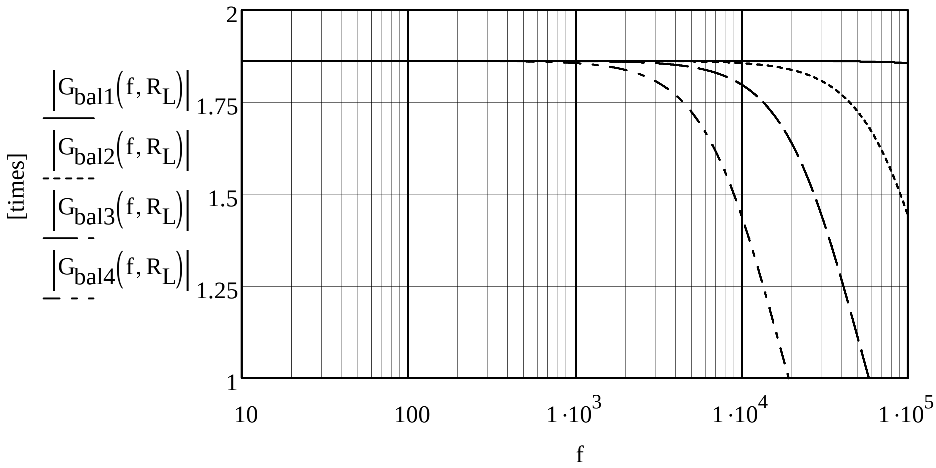

Fig. 10.15 Balanced gain vs. frequency with equal additional capacitances parellel to the anode and cathode load resistors R

$k := 1..5 \qquad f := 20\cdot10^3 Hz \qquad C_{p_k} :=$

$10\cdot10^{-12}F$
$1\cdot10^{-9}F$
$10\cdot10^{-9}F$
$33\cdot10^{-9}F$
$100\cdot10^{-9}F$

$$Z_k := \left(\frac{1}{R}+2j\cdot\pi\cdot f\cdot C_{p_k}\right)^{-1} \qquad G_{bal_k} := \frac{2\cdot\mu}{2+\mu+r_a\cdot\left(\frac{1}{Z_k}+\frac{1}{0.5\cdot R_L}\right)}$$

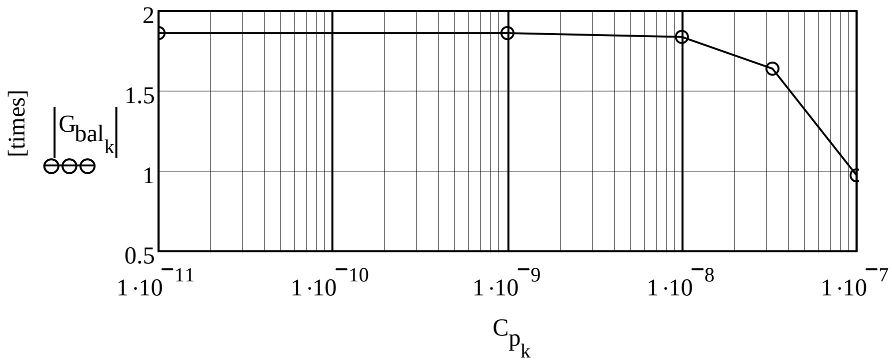

Fig. 10.15 Balanced gain at a f = 20kHz vs. various equal additional capacitances parallel to the anode and cathode load resistors R

2. CPS used as un-balanced to un-balanced converter via anode OR cathode output

2.1 Idle gains

via the anode alone: $G_{0.a} := -\mu \cdot \dfrac{R_a}{r_a + R_a + (1+\mu) \cdot R_c}$ $\qquad G_{0.a} = -931.293 \times 10^{-3}$

via the cathode alone: $G_{0.c} := \mu \cdot \dfrac{R_c}{r_a + R_a + (1+\mu) \cdot R_c}$ $\qquad G_{0.c} = 931.293 \times 10^{-3}$

2.2 Output resistances (between respective output and ground!)

via the anode alone: $R_{o.a} := \dfrac{R_a \cdot \left[r_a + (1+\mu) \cdot R_c\right]}{r_a + R_a + (1+\mu) \cdot R_c}$ $\qquad R_{o.a} = 14.549 \times 10^{3}\ \Omega$

via the cathode alone: $r_c := \dfrac{r_a + R_a}{1+\mu}$ $\qquad R_{o.c} := \dfrac{R_c \cdot r_c}{r_c + R_c}$ $\qquad R_{o.c} = 579.972 \times 10^{0}\ \Omega$

2.3 Output loaded gains (outputs separately loaded!)

$R_{L.a} := 5 \cdot 10^{3} \Omega$ $\qquad R_{L.c} := 5 \cdot 10^{3} \Omega$

$G_a(R_{L.a}) := G_{0.a} \cdot \dfrac{R_{L.a}}{R_{L.a} + R_{o.a}}$ $\qquad G_a(R_{L.a}) = -238.19 \times 10^{-3}$

$G_c(R_{L.c}) := G_{0.c} \cdot \dfrac{R_{L.c}}{R_{L.c} + R_{o.c}}$ $\qquad G_c(R_{L.c}) = 885.48 \times 10^{-3}$

3. Derivation of the balanced equations and proof of the above given ones

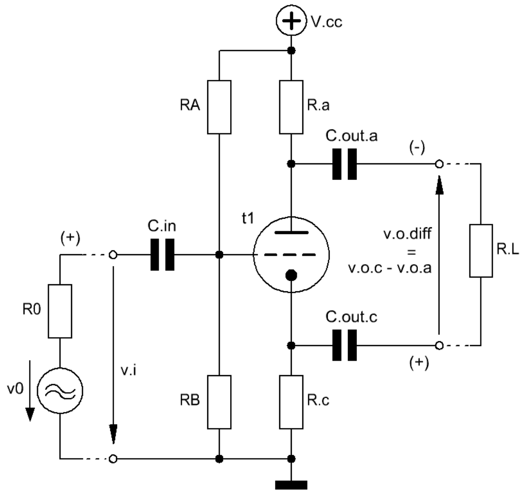

Fig. 10.16 Un-balanced to balanced concerter

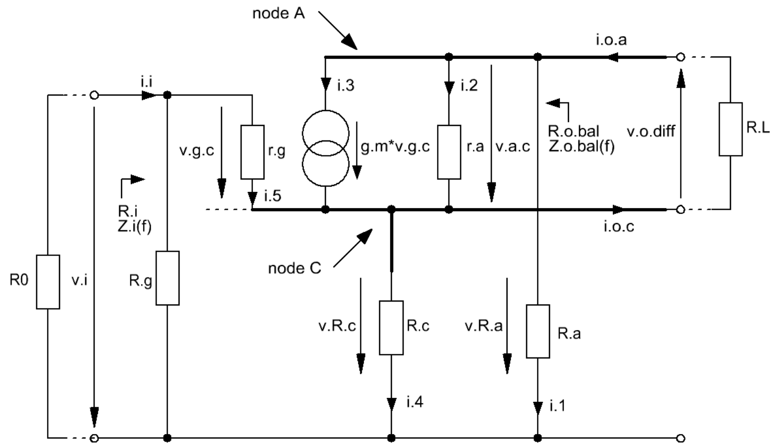

Fig. 10.17 Equivalent circuit of Fig. 10.16

3.1 Relevant derivation equations

node A: $i_1 + i_2 + i_3 = i_{o.a}$

node C: $i_2 + i_3 = i_{o.c} + i_4$ $\quad i_{o.a} = i_{o.c}$ $\quad$ => $\quad i_1 + i_2 + i_3 = i_2 + i_3 - i_4$ $\quad$ => $\quad i_1 = -i_4$

$R_a = R_c = R$ $\quad$ => $\quad v_{R.c} = -v_{R.a}$

$i_1 = \frac{v_{R.a}}{R}$ $\quad i_2 = \frac{v_{a.c}}{r_a}$ $\quad i_3 = g_m \cdot v_{g.c}$ $\quad i_4 = \frac{v_{R.c}}{R}$ $\quad i_{o.c} = i_{o.a} = \frac{v_{o.diff}}{R_L}$

$v_{o.diff} = -v_{a.c}$ $\quad v_{g.c} = v_i - v_{R.c}$ $\quad v_{R.c} + v_{a.c} = v_{R.a}$ $\quad v_{a.c} = v_{R.a} - v_{R.c} = -v_{o.diff} = -2 \cdot v_{R.c}$ $\quad v_{R.c} = \frac{v_{o.diff}}{2}$

=> $$\frac{v_{R.a}}{R} + \frac{v_{a.c}}{r_a} + g_m \cdot v_{gc} = \frac{v_{o.diff}}{R_L}$$

=> $$\frac{v_{a.c}}{r_a} + g_m \cdot v_{gc} = \frac{v_{o.diff}}{R_L} + \frac{v_{R.c}}{R}$$

3.2 Derivation of the differential (= balanced) gain with output loaded (see Fig. 10.17)

=> $$\frac{-v_{o.diff}}{r_a} + g_m \cdot v_i - g_m \cdot \frac{v_{o.diff}}{2} = \frac{v_{o.diff}}{R_L} + \frac{v_{o.diff}}{2 \cdot R}$$

$$v_{o.diff} \cdot \left(-\frac{1}{r_a} - \frac{g_m}{2} - \frac{1}{R_L} - \frac{1}{2 \cdot R}\right) = -v_i \cdot g_m \Rightarrow \frac{v_{o.diff}}{v_i} = G_{diff}(R_L) = -g_m \cdot \left(-\frac{1}{r_a} - \frac{g_m}{2} - \frac{1}{R_L} - \frac{1}{2 \cdot R}\right)^{-1}$$

$$\frac{v_{o.diff}}{v_i} = G_{diff}(R_L) = \frac{R \cdot 2 \cdot g_m \cdot r_a \cdot R_L}{2 \cdot R_L \cdot R + g_m \cdot r_a \cdot R_L \cdot R + 2 \cdot r_a \cdot R + r_a \cdot R_L}$$

$R := 15 \cdot 10^3 \Omega$ $\qquad$ $R_L := 10 \cdot 10^3 \Omega$

$$\frac{R \cdot 2 \cdot g_m \cdot r_a \cdot R_L}{2 \cdot R_L \cdot R + g_m \cdot r_a \cdot R_L \cdot R + 2 \cdot r_a \cdot R + r_a \cdot R_L} = 1.816 \times 10^0$$

$$G_{diff1}(R_L) := \frac{2 \cdot \mu \cdot R \cdot R_L}{(2 + \mu) \cdot (R_L \cdot R) + r_a \cdot (2 \cdot R + R_L)}$$

$G_{diff1}(R_L) = 1.816 \times 10^0$

=> $$G_{diff}(R_L) := \frac{2 \cdot \mu}{2 + \mu + r_a \cdot \left(\frac{1}{R} + \frac{1}{0.5 \cdot R_L}\right)}$$

$G_{diff}(R_L) = 1.816 \times 10^0$

=> $G_{diff}(R_L) = G_{bal}(R_L)$ $\qquad$ **q.e.d**

with R_L = infinite: $$G_{0.diff} := \frac{2 \cdot \mu}{(2 + \mu) + \frac{r_a}{R}}$$

$G_{0.diff} = 1.863 \times 10^0$

=> $G_{0.diff} = G_{0.bal}$ $\qquad$ **q.e.d**

3.3 Derivation of the differential (= balanced) output resistance (see Fig. 10.17)

3.3.1 Equations approach

$i_{o.a} = i_1 + i_2 + i_3$ $v_{o.diff} = -i_2 \cdot r_a$ $i_2 = \frac{-v_{o.diff}}{r_a}$ $i_3 = g_m \cdot v_{g.c}$ $i_1 = \frac{v_{R.a}}{R}$

$$i_{o.a} = \frac{v_{R.a}}{R} - \frac{v_{o.diff}}{r_a} + g_m \cdot v_{g.c} = \frac{-v_{o.diff}}{2 \cdot R} - \frac{v_{o.diff}}{r_a} + g_m \cdot v_i - \frac{g_m}{2} \cdot v_{o.diff}$$

with $v_i = 0$ => $i_{o.a} = v_{o.diff} \cdot \left(-\frac{1}{2 \cdot R} - \frac{1}{r_a} - \frac{g_m}{2} \right)$

$$R_{o.diff} = \frac{-v_{o.diff}}{i_{o.a}} = \frac{-v_{o.diff}}{i_{o.c}} = \frac{-v_{o.diff}}{v_{o.diff} \cdot \left(-\frac{1}{2 \cdot R} - \frac{1}{r_a} - \frac{g_m}{2} \right)} = \frac{1}{\frac{1}{2 \cdot R} + \frac{1}{r_a} + \frac{g_m}{2}}$$

$$R_{o.diff1} := \frac{1}{\frac{1}{2 \cdot R} + \frac{g_m}{2} + \frac{1}{r_a}}$$ $R_{o.diff1} = 258.693 \times 10^0\ \Omega$

=> $R_{o.diff} := \frac{2 \cdot r_a \cdot R}{r_a + (2 + \mu) \cdot R}$ $R_{o.diff} = 258.693 \times 10^0\ \Omega$

=> $R_{o.diff} = R_{o.bal}$ **q.e.d**

3.3.2 '0.5-times' approch

At the gain $G_{bal.L} = 0.5*G_{bal.nL}$ the output resistance $R_{o.bal}$ equals the load resistance R_L. Hence,

$0.5 = \frac{G_{bal}(R_L)}{G_{0.bal}}$ => $0.5 = \frac{\frac{2 \cdot \mu}{2+\mu+r_a \cdot \left(\frac{1}{R} + \frac{1}{0.5 \cdot R_L} \right)}}{\frac{2 \cdot \mu}{(2+\mu)+\frac{r_a}{R}}}$

=> $R_L := \frac{2 \cdot r_a \cdot R}{r_a + (2 + \mu) \cdot R}$ => $R_L = 258.693 \times 10^0\ \Omega$

with $R_{o.bal} := R_L$ => $R_{o.bal} = 258.693 \times 10^0\ \Omega$ **q.e.d**