Evaluation of Various Window Functions using Multi-Instrument

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1. Spectral Leakage and Window Function

Spectral leakage is the result of the assumption in the FFT algorithm that the time record in a FFT segment is exactly repeated throughout all time and that signals contained in a FFT segment are thus periodic at intervals that correspond to the length of the FFT segment. If the time record in a FFT segment has a non-integer number of cycles, this assumption is violated and spectral leakage occurs. Spectral leakage distorts the measurement in such a way that energy from a given frequency component spreads to adjacent frequency lines or bins. In most cases, you cannot guarantee that you are sampling an integer number of cycles in a FFT segment. Choosing a window function correctly to suppress the spectral leakage for a certain measurement is thus critical.

To choose a window function, you must estimate the signal frequency content first. If the signal contains strong interfering frequency components distant from the frequency of interest, choose a window with a high side lobe roll-off rate. If there are strong interfering signal near the frequency of interest, choose a window with a low highest side lobe level. If the frequency of interest contains two or more signals very near to each other, then frequency resolution is very important. It is best to choose a window with a very narrow main lobe. If the amplitude accuracy of a single frequency component is more important than the exact location of the component in a given frequency bin, choose a window with a wide main lobe. If the signal spectrum is rather flat or broadband in frequency content, use the Rectangle Window. In general, the Hann Window has good frequency resolution and reduced spectral leakage. It is satisfactory in 95% of cases.

For more information on FFT basics, please refer to:

http://www.virtins.com/doc/D1002/FFT_Basics_and_Case_Study_using_Multi-Instrument_D1002.pdf

2. How to use Multi-Instrument to evaluate a Window Function

Multi-Instrument is a powerful multi-function virtual instrument software. It supports a variety of hardware ranging from sound cards which are available in almost all computers to proprietary ADC and DAC hardware such as NI DAQmx cards, VT DSO, VT RTA, VT IEPE, VT CAMP and so on. The software can be downloaded at: www.virtins.com/MIsetup.exe and www.multi-instrument.com/MIsetup.exe for 21-day fully functional FREE trial.

2.1 Method 1: Analyze the window function WAV file with no window function

Multi-Instrument supports 60 window functions. A 1024-point 24-bit WAV file of each window function is provided in the “WAV\window” directory of the software and can be used to evaluate the behavior of each window function in the frequency domain. The

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sampling rate of these WAV files is 44100 Hz.

To evaluate a window function using the provided WAV file, the following changes to the system default settings for the Spectrum Analyzer are required after loading the WAV file via [File]>[Open]:

- [Window]: Rectangle;
- [FFT size]: >1024 (32768 is enough to show the shape of the spectrum of the window function, but 4194304 will be used in this article to obtain more accurate window parameters such as –3dB Main Lobe Width);
- [Setting]>[Spectrum Analyzer Processing]>[Intra-Frame Processing] > [Remove DC]: Unchecked;

The following changes to the system default settings in the Spectrum Analyzer are recommended:

- [Setting]>[Spectrum Analyzer Y Scale]: dBr
- [Horizontal Axis Multiplier]: ×20

This method will be used in this article to evaluate those window functions that has the highest side lobe level greater than -145dB (i.e. those window functions from Rectangle to Kaiser 6 in Multi-Instrument). From Kaiser 7 to Cosine Sum 261, the difference between the main lobe and the highest side lobe is more than 145dB. The required numerical precision for a varying signal cannot be kept in a 24-bit WAV file and thus the following method will be used instead.

2.2 Method 2: Analyze a unit DC signal with the window function to be evaluated

The 1024-point 24-bit WAV file of the Rectangle window in the “WAV\window” directory of the software can be used as the unit DC signal. To use it to evaluate a window function, the following changes to the system default settings for the Spectrum Analyzer are required after loading the WAV file via [File]>[Open]:

- [Window]: the window function to be evaluated;
- [FFT size]: > 1024;
- [Setting]>[Spectrum Analyzer Processing]>[Intra-Frame Processing] > [Remove DC]: Unchecked;

The following changes to the system default settings in the Spectrum Analyzer are recommended:

- [Setting]>[Spectrum Analyzer Y Scale]: dBr
- [Horizontal Axis Multiplier]: ×20

One advantage of this method is that the real window function used in the software is evaluated with high precision. The drawback is that the shape of the window function is not shown in the Oscilloscope and some of the parameters which are calculated from the

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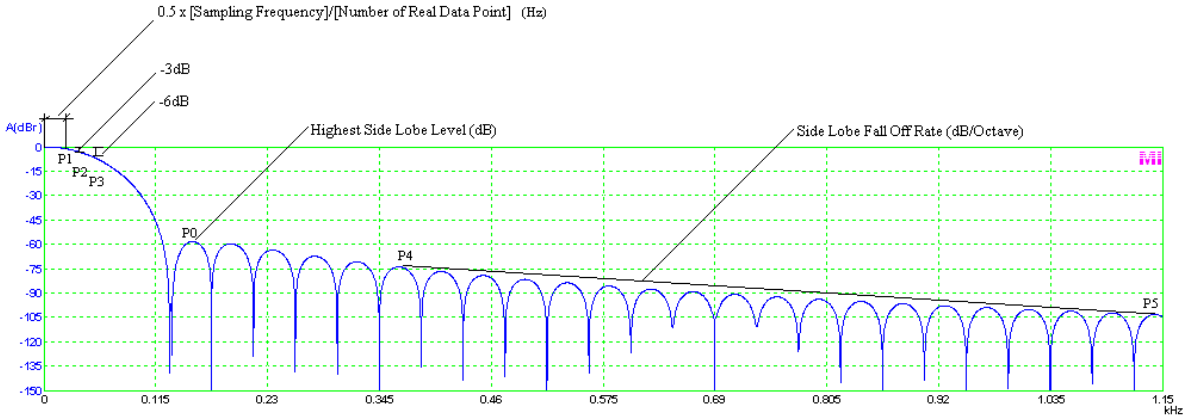
Oscilloscope (e.g. Coherent Gain and Equivalent Noise Bandwidth) cannot be obtained. They will be obtained using Method 1 instead without compromising the precision.

2.3 Window Function Parameters to Be Evaluated

The spectrum of a window function is continuous with a main lobe and several side lobes (see figure below). The width of the main lobe limits the frequency resolution of a windowed signal. The ability to distinguish two closely spaced frequency components increases as the width of the main lobe of the window function decreases. The width of the main lobe can be described by its width at –3 dB and –6 dB below the peak of the main lobe. The side lobes are characterized by the Highest Side Lobe Level and the Side Lobe Fall Off Rate.

There are some more abstract parameters for a window functions, such as Scallop Loss, Coherent Gain, Equivalent Noise Bandwidth, etc. Readers are recommended to reference to the classic paper "On the use of Windows for Harmonic Analysis with the Discrete Fourier Transform", Fredric J. Harris, *Proceedings of The IEEE*, Vol. 66, No. 1, January 1978.



The following parameters of each window function can be measured with the help of the cursor reader provided in Multi-Instrument. (see figure above)

- Highest Side Lobe Level (dB)
  $= y_{P0}$
  where P0 is located at the highest peak of the side lobes and y_{P0} is its Y value in dB.

- Side Lobe Fall Off Rate (dB/Octave)
  $= \log(2) \times (y_{P5} - y_{P4}) / [\log(n_{P5}) - \log(n_{P4})]$
  where y_{P4} and y_{P5} are the Y value in dB for P5 and P6 respectively, and n_{P4} and n_{P5} are the X value in count for P5 and P6 respectively, n_{P4}, $n_{P5} = 0, 1, \ldots, N$, where N is the FFT size.

- -3dB Main Lobe Width (bins)
  $= 2 \times n_{P2} \times N_{original} / N$
  where P2 is located at Y= -3 dB within the main lobe, and n_{P2} is its X value in count, $n_{P2} = 0, 1, \ldots, N$, where N is the FFT size. $N_{original}$ is the number of original data point before zero padding. In the examples of this article, $N_{original} = 1024$ and $N = 4194304$.

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- -6dB Main Lobe Width (bins)
  $= 2 \times n_{P3} \times N_{original} / N$
  where P3 is located at Y= -6 dB within the main lobe, and n_{P3} is its X value in count. n_{P3} = 0, 1, …, N, where N is the FFT size. $N_{original}$ is the number of original data point before zero padding. In the examples of this article, $N_{original}$ = 1024 and N = 4194304.

- Scallop Loss (dB)
  $= -y_{P1}$
  where P1 is located at $X = 0.5 \times N / N_{original}$, y_{P1} is the Y value in dB for P1. $N_{original}$ is the number of original data point before zero padding, and N is the FFT size. In the examples of this article, $N_{original}$ = 1024 and N = 4194304.

The following parameters of each window function can be obtained via the mean and RMS values measured in Multi-Instrument.

- Coherence Gain
  = Mean
  where Mean is the mean value of the window function waveform.

- Equivalent Noise Bandwidth
  $= RMS^2 / Mean^2$
  where RMS and Mean are the RMS and mean values of the window function waveform respectively.

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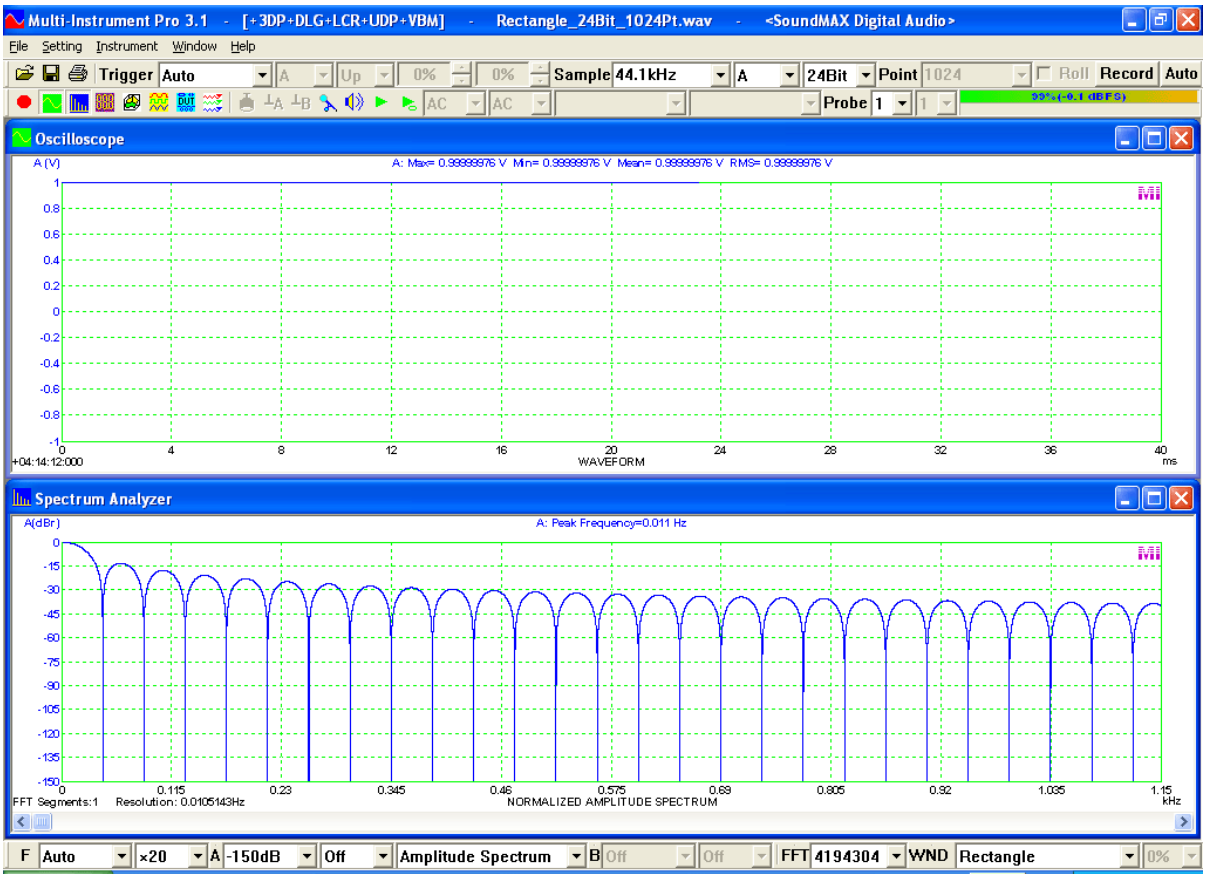
3. Window Functions

3.1 Rectangle Window

w(n) = 1, n = 0, 1, …, N-1

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-13	-6	0.88	1.21	3.92	1	1



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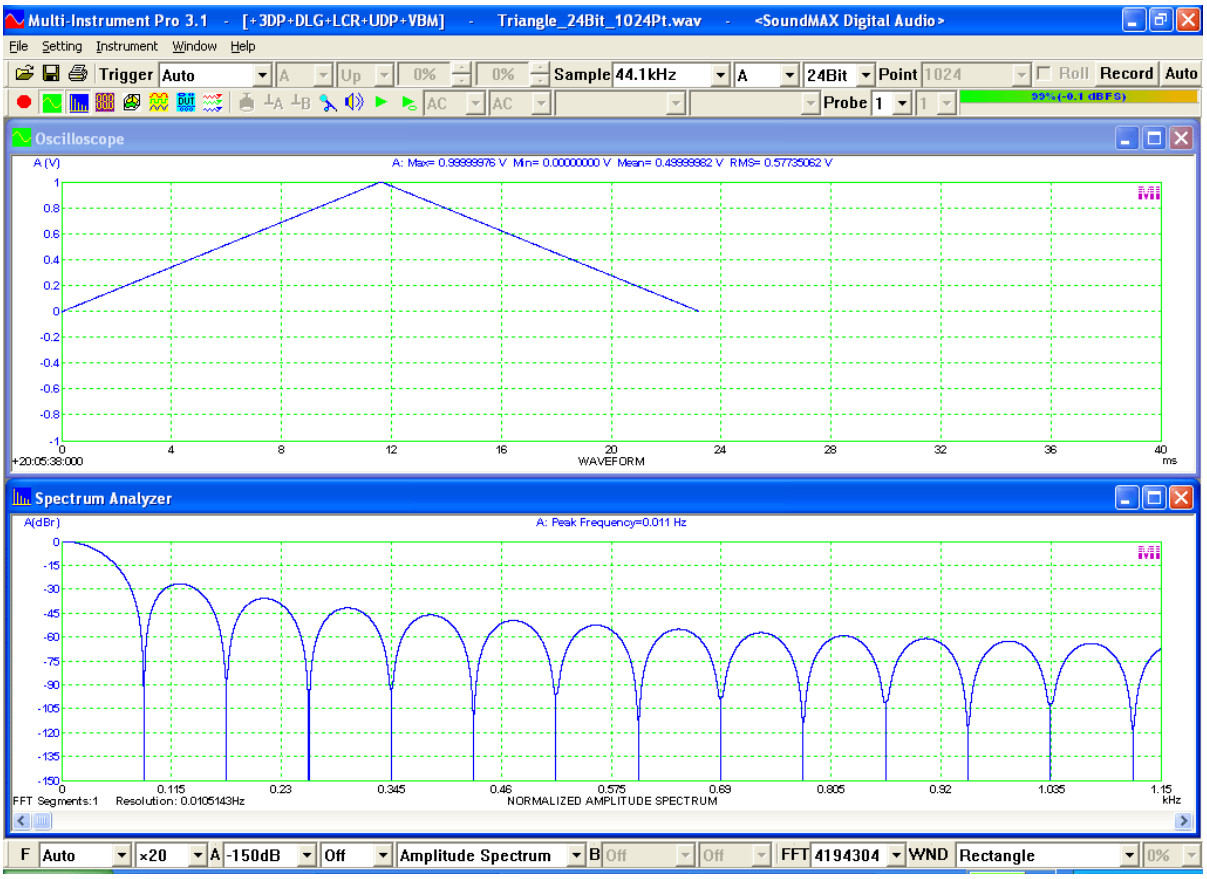
3.2 Triangle Window

w(n) = n/(N/2), n = 0,1,…,N/2;
w(n) = w(N-n), n = N/2,…,N-1;

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-27	-12	1.28	1.78	1.82	0.5	1.33



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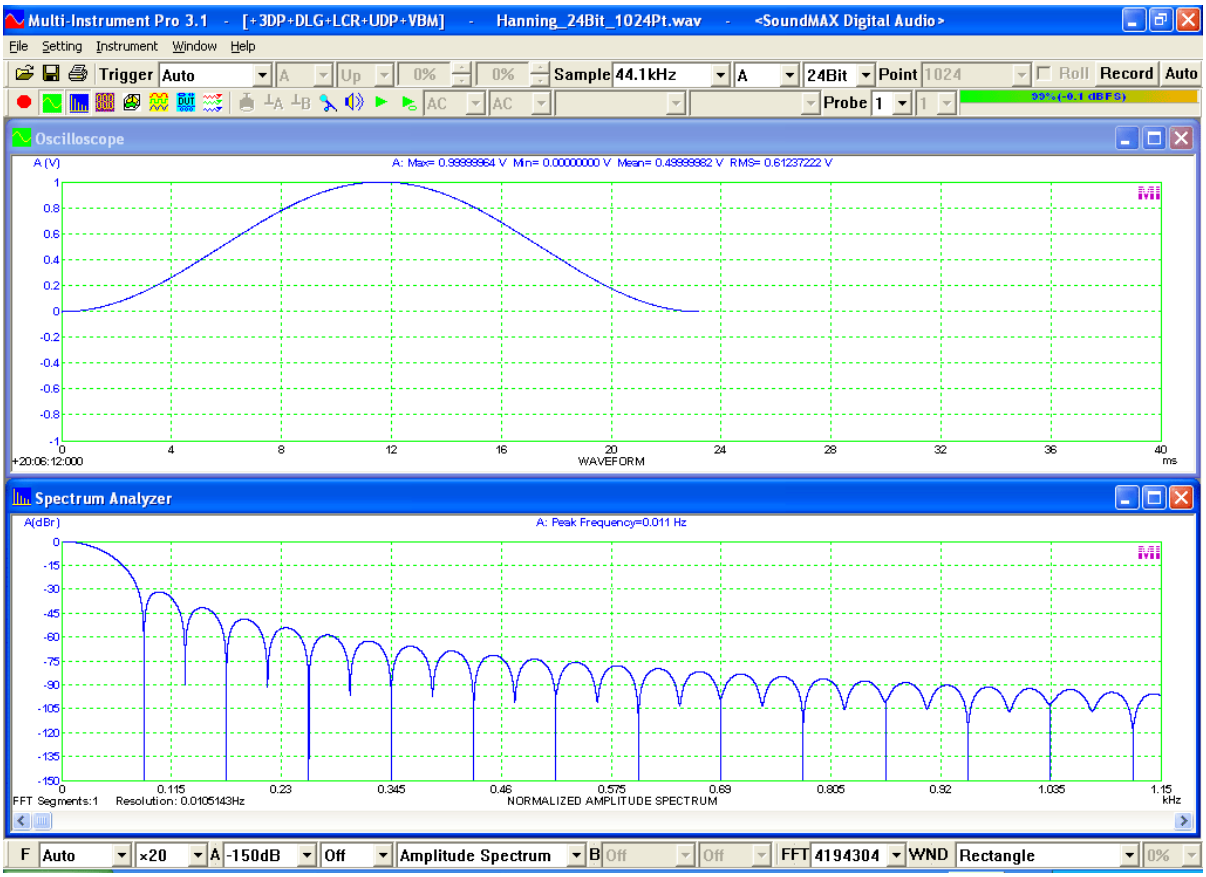
3.3 Hanning Window

$w(n) = \sin^2(n\pi/N) = 0.5-0.5\cos(2n\pi/N), \quad n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-32	-18	1.44	2.00	1.42	0.5	1.50



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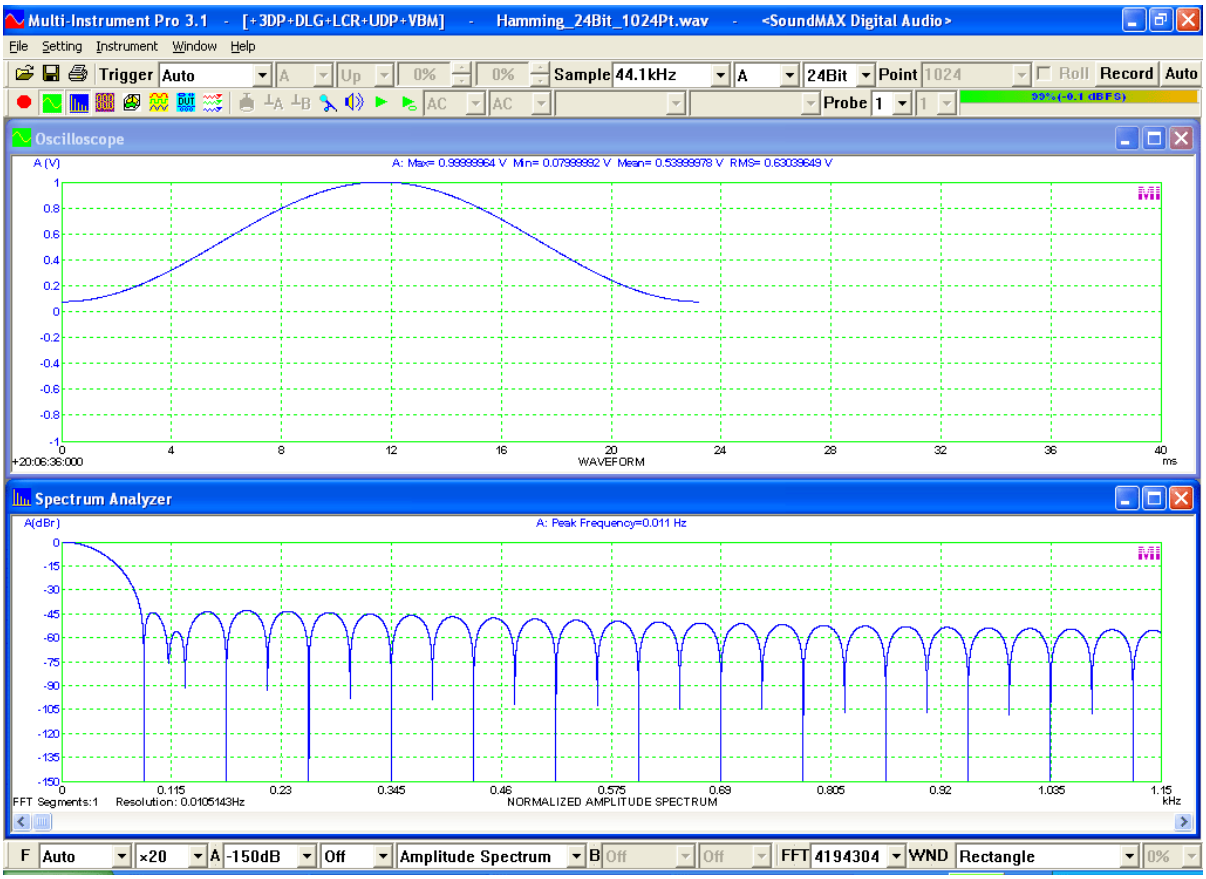
3.4 Hamming Window

$w(n) = 0.54 – 0.46\cos(2n\pi/N), \quad n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-43	-6	1.30	1.81	1.75	0.54	1.36



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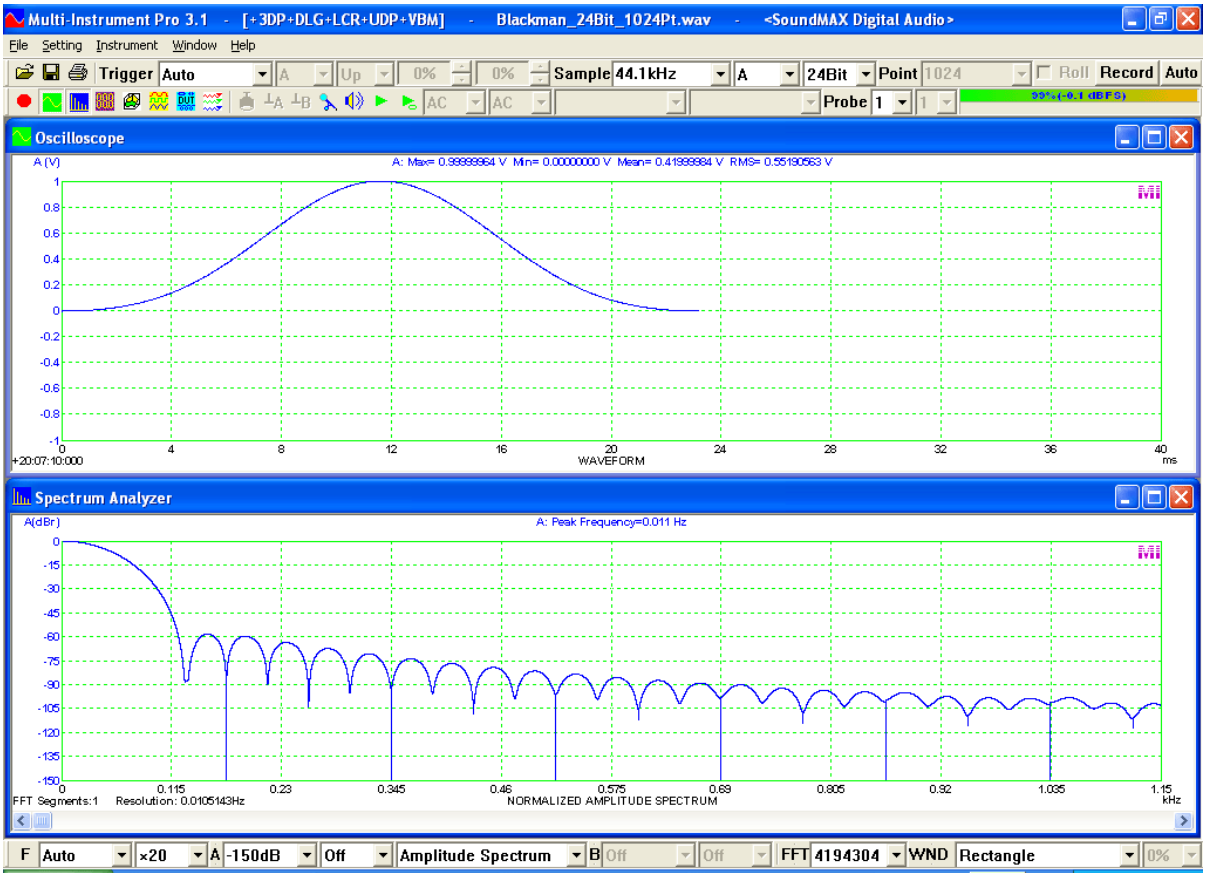
3.5 Blackman Window

$w(n) = 0.42 – 0.5\cos(2n\pi/N) + 0.08\cos(4n\pi/N), \quad n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-58	-18	1.64	2.29	1.10	0.42	1.73



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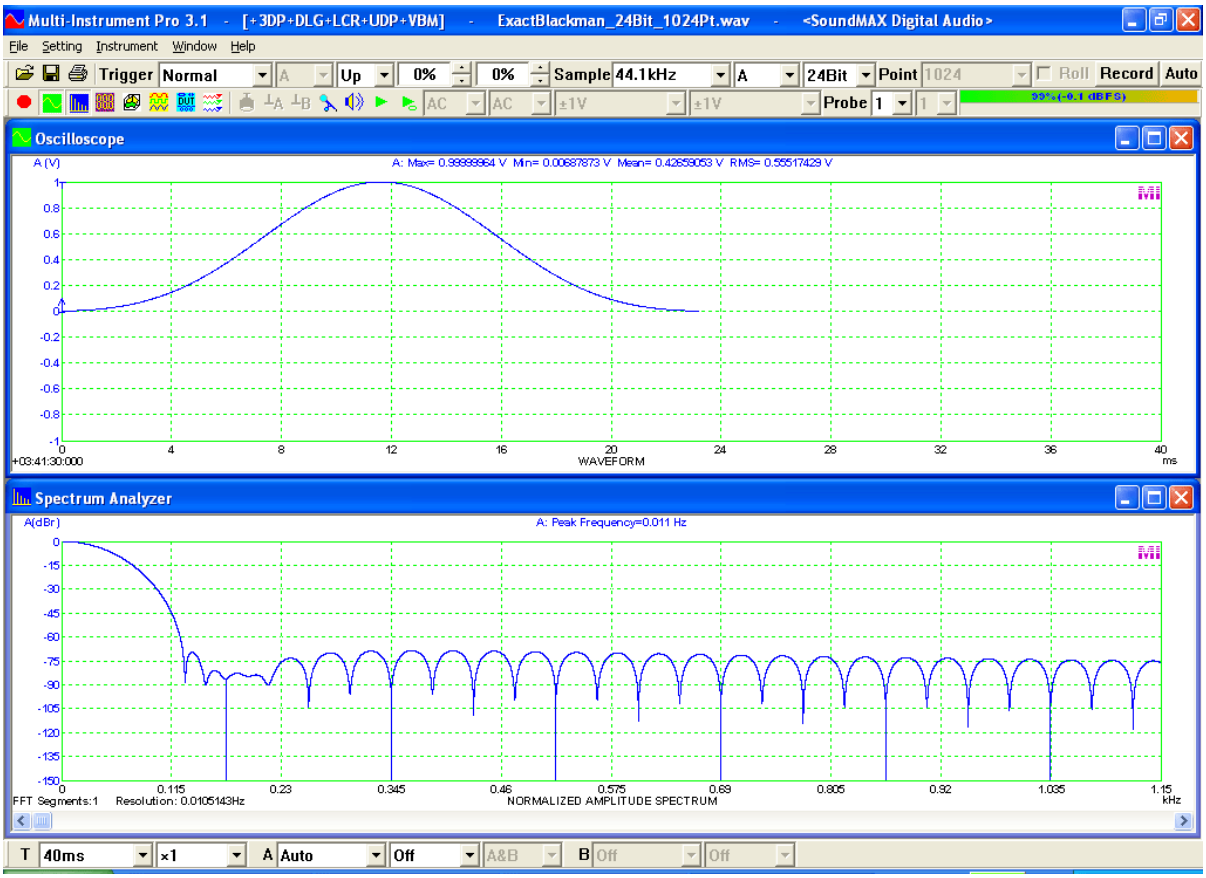
3.6 Exact Blackman Window

$$w(n) = 7938/18608 – 9240/18608 \times \cos(2n\pi/N) + 1430/18608 \times \cos(4n\pi/N)$$

$$n = 0,1,\ldots,N-1;$$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-68	-6	1.60	2.25	1.15	0.43	1.69



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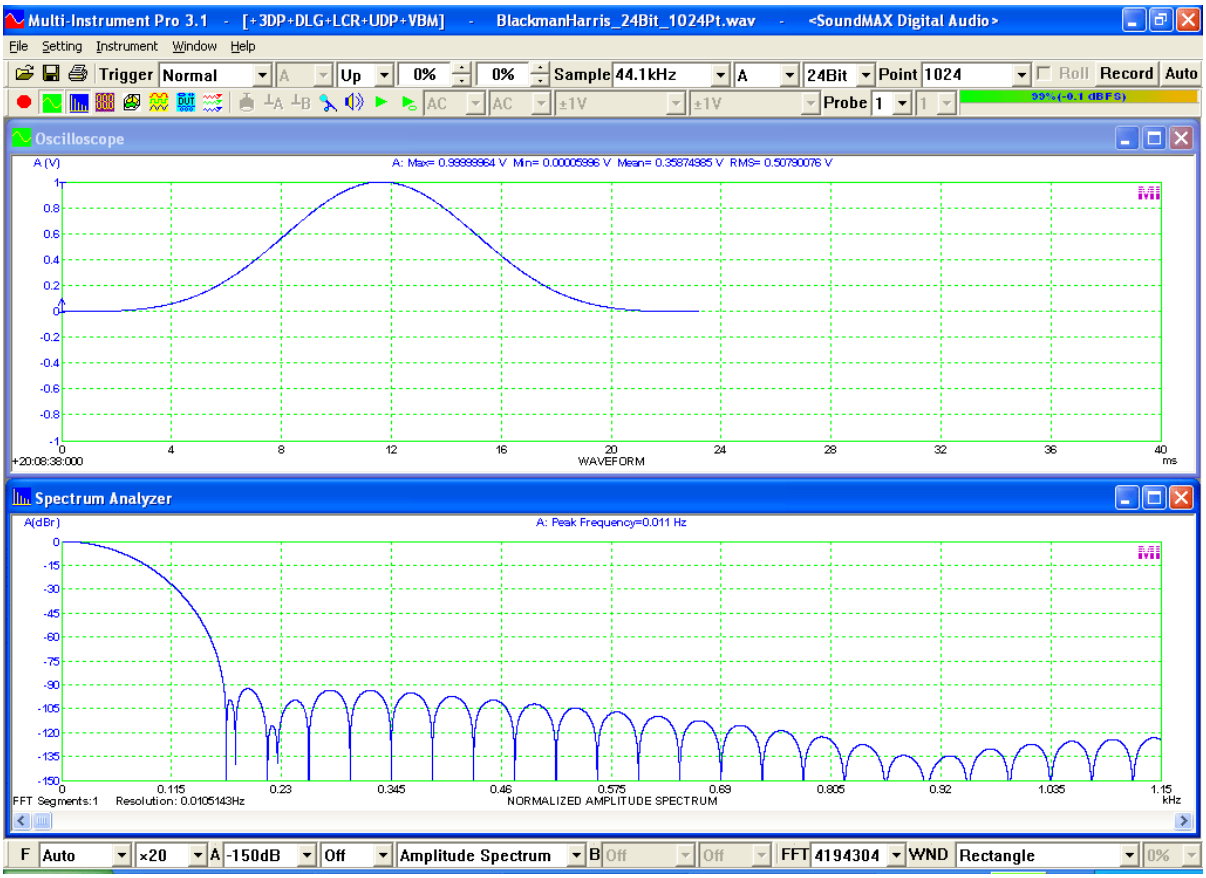
3.7 Blackman-Harris (4 terms) Window

$w(n) = 0.35875 – 0.48829 \times \cos(2n\pi/N) + 0.14128 \times \cos(4n\pi/N) –0.01168 \times \cos(6n\pi/N)$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-92	-6	1.90	2.66	0.83	0.36	2.00



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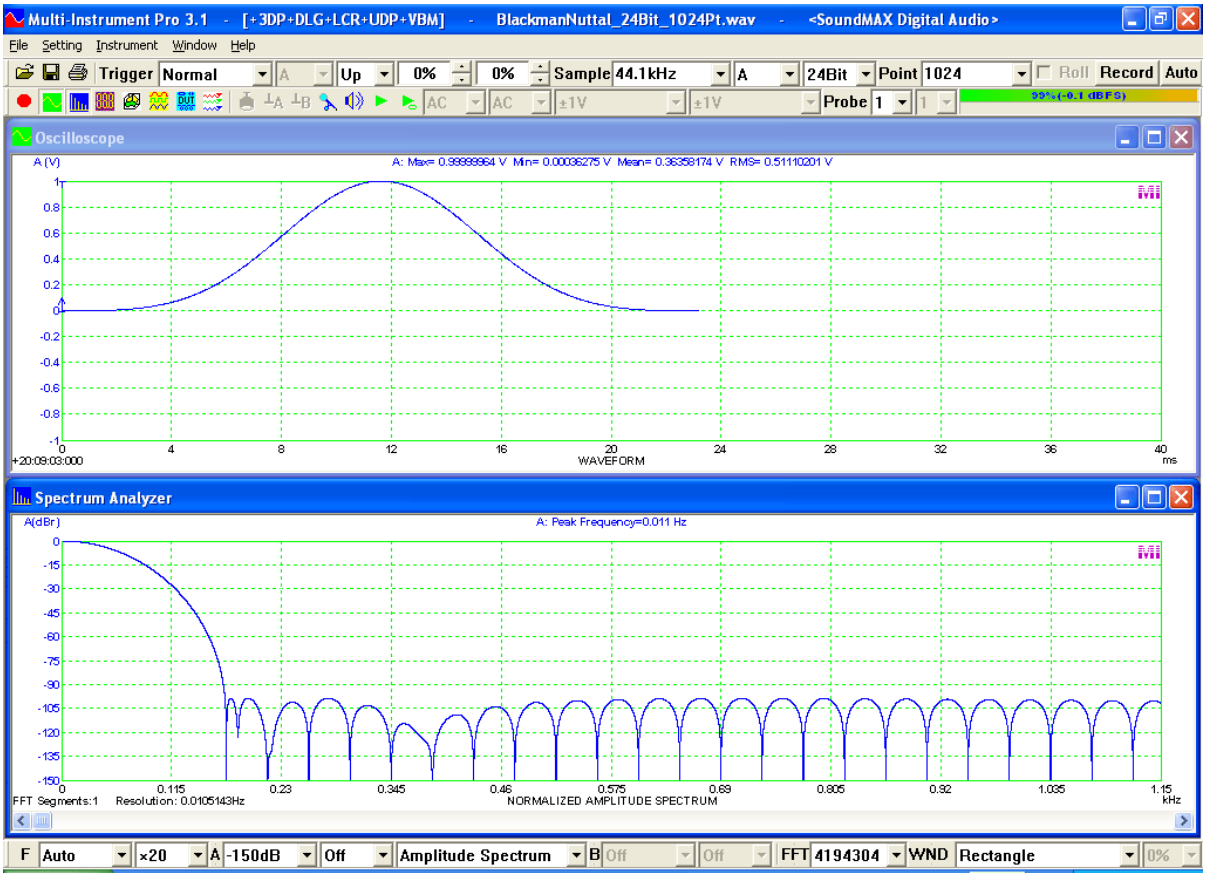
3.8 Blackman-Nuttall Window

$w(n) = 0.3635819 – 0.4891775 \times \cos(2n\pi/N) + 0.1365995 \times \cos(4n\pi/N) –0.0106411 \times \cos(6n\pi/N)$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-98	-6	1.87	2.63	0.85	0.36	1.98



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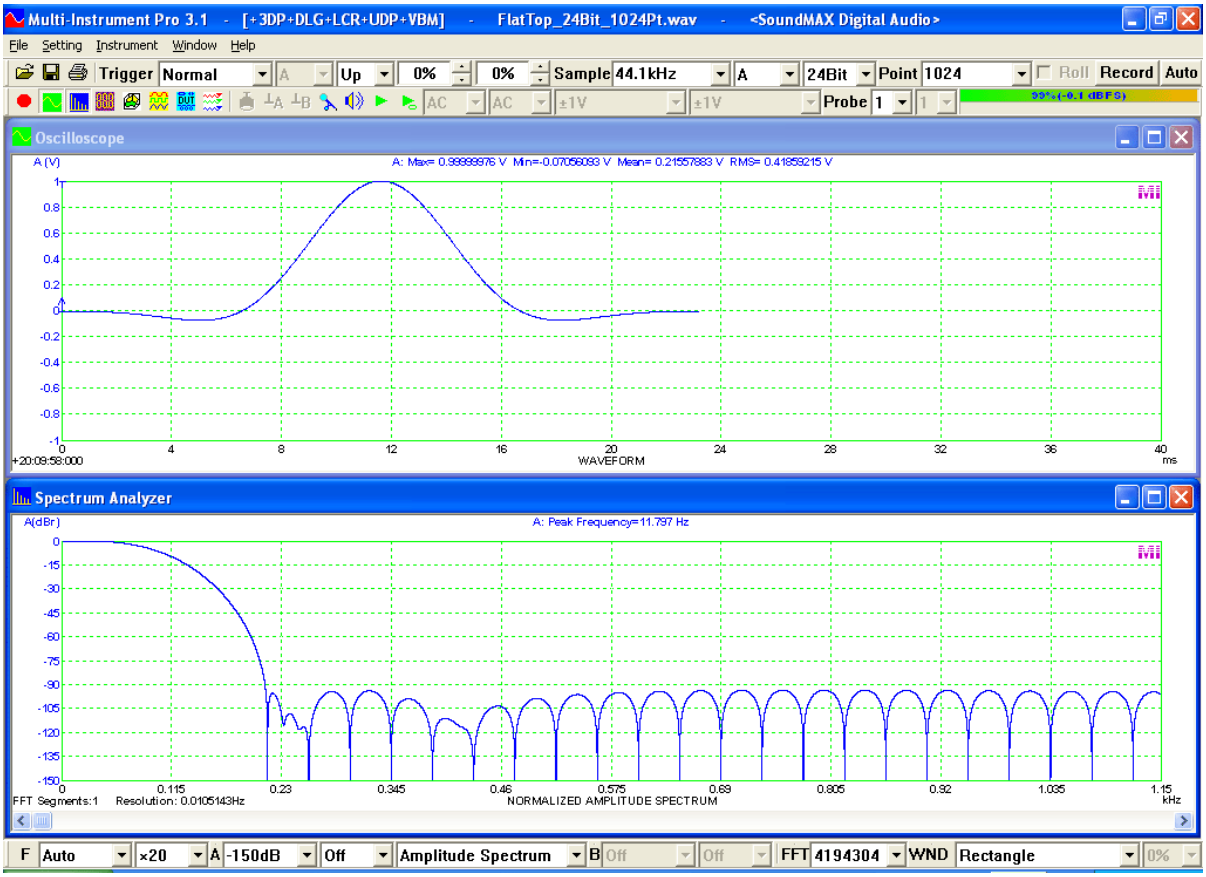
3.9 Flat Top Window

$$w(n) = 0.21557895 - 0.41663158 \times \cos(2n\pi/N) + 0.277263158 \times \cos(4n\pi/N) - 0.083578947 \times \cos(6n\pi/N) + 0.006947368 \times \cos(8n\pi/N)$$

$$n = 0,1,\ldots,N-1;$$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-94	-6	3.72	4.58	0.012	0.22	3.77



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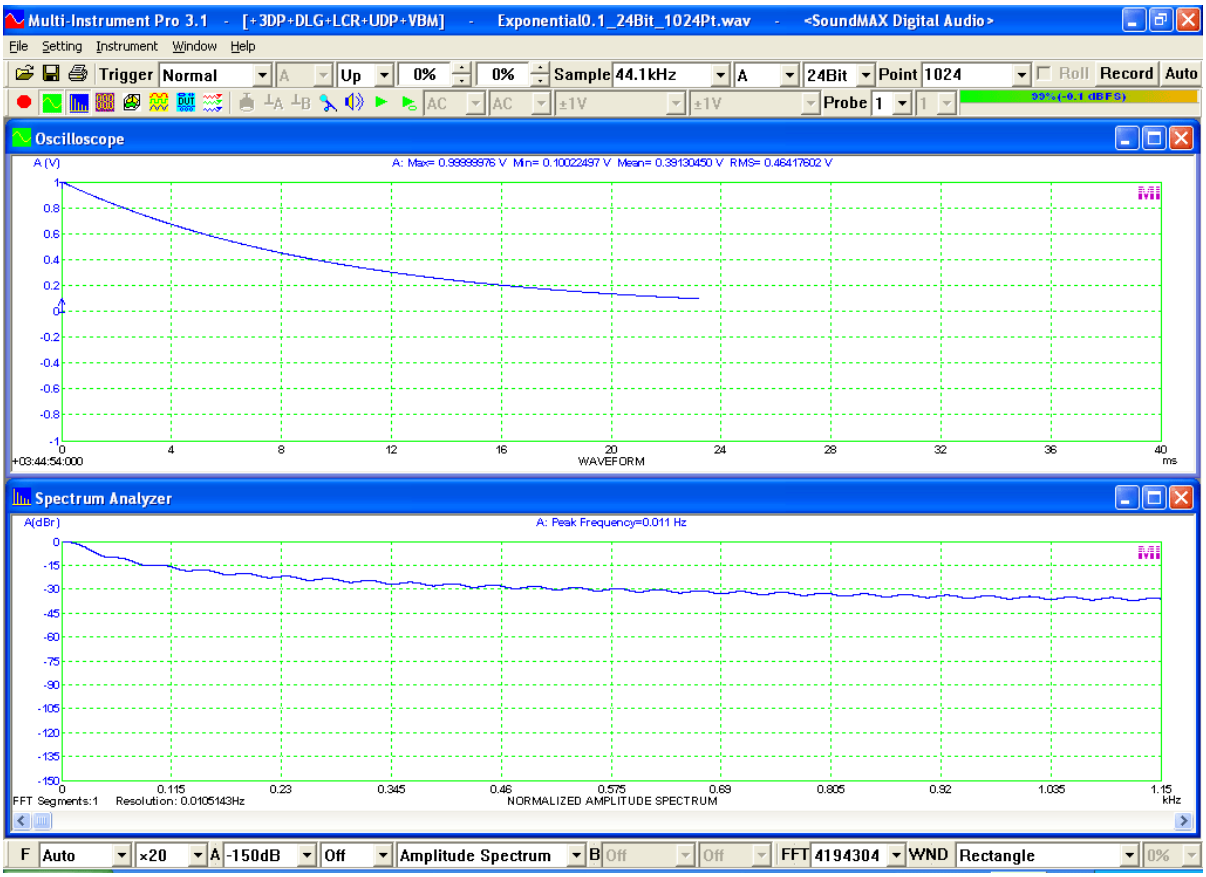
3.10 Exponential Window (α = 0.1)

$w(n) = \alpha^{n/N}$, $0<\alpha<1$, $n = 0,1,\ldots,N-1$;

$\alpha = 0.1$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-10	-6	1.03	1.47	2.83	0.39	1.41



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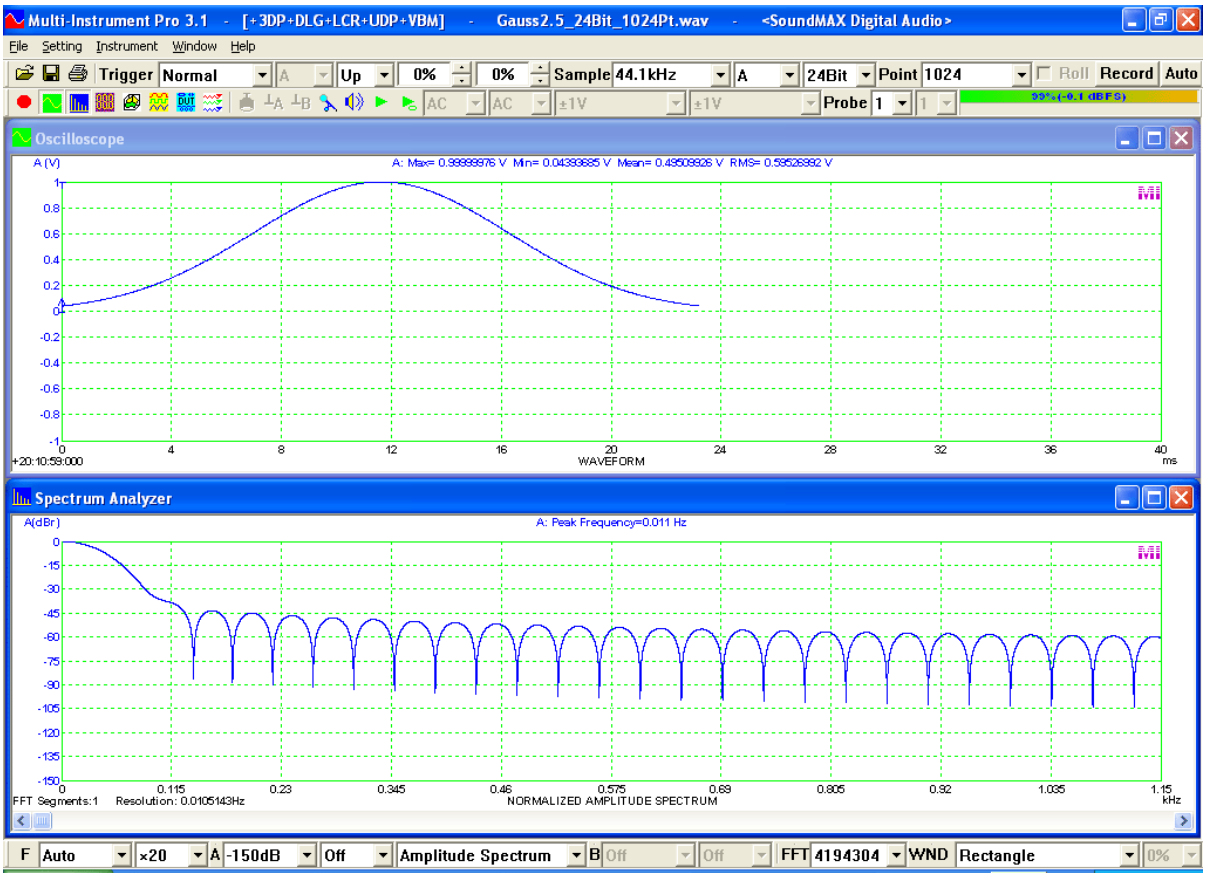
3.11 Gaussian Window ($\alpha = 2.5$)

$w(n) = e^{-0.5[\alpha\,(2n/N-1)]\,[\alpha\,(2n/N-1)]}$, $\alpha >= 2$, $n = 0,1,\ldots,N-1$;

$\alpha = 2.5$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-43	-6	1.37	1.92	1.58	0.50	1.45



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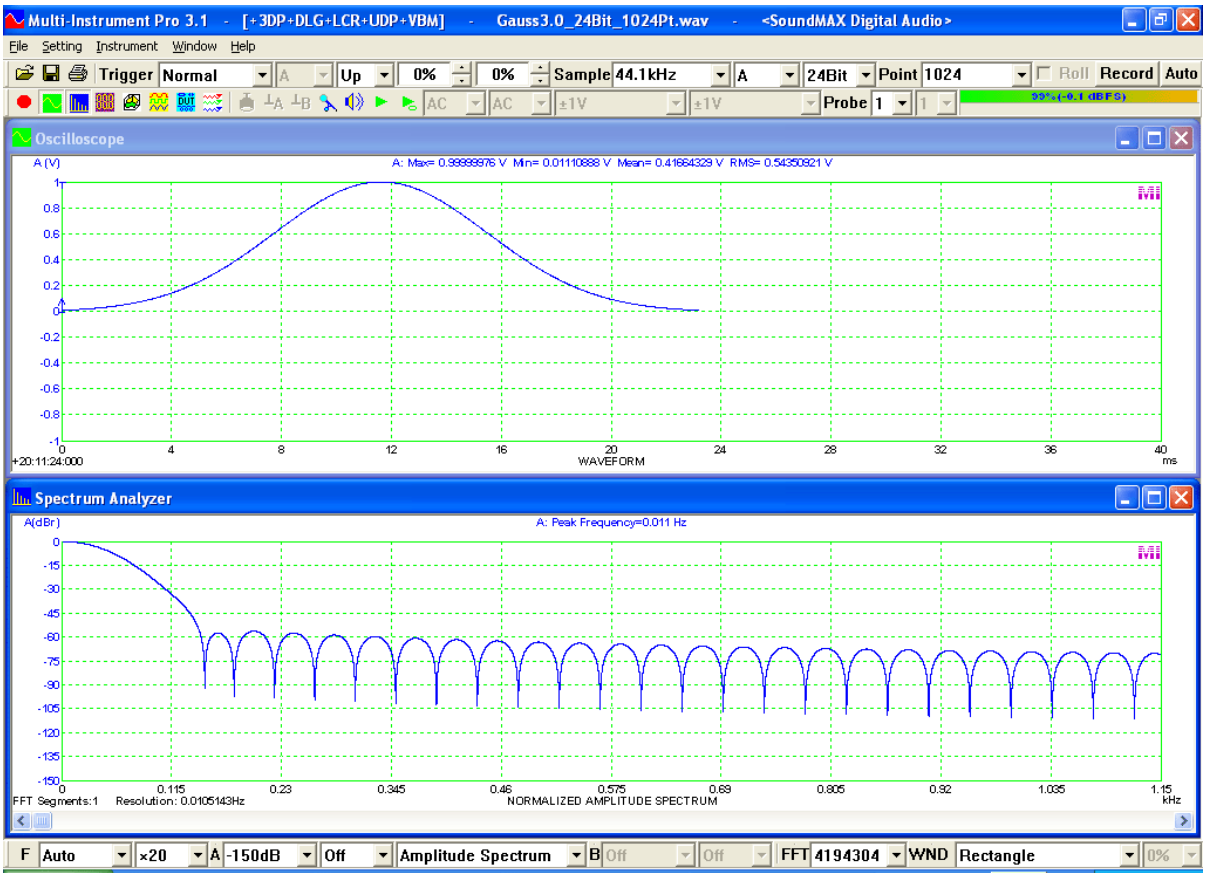
3.12 Gaussian Window ($\alpha = 3.0$)

$w(n) = e^{-0.5[\alpha\,(2n/N-1)]\,[\alpha\,(2n/N-1)]}, \quad \alpha >= 2, \quad n = 0,1,\ldots,N-1;$

$\alpha = 3.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-56	-6	1.60	2.26	1.16	0.42	1.71



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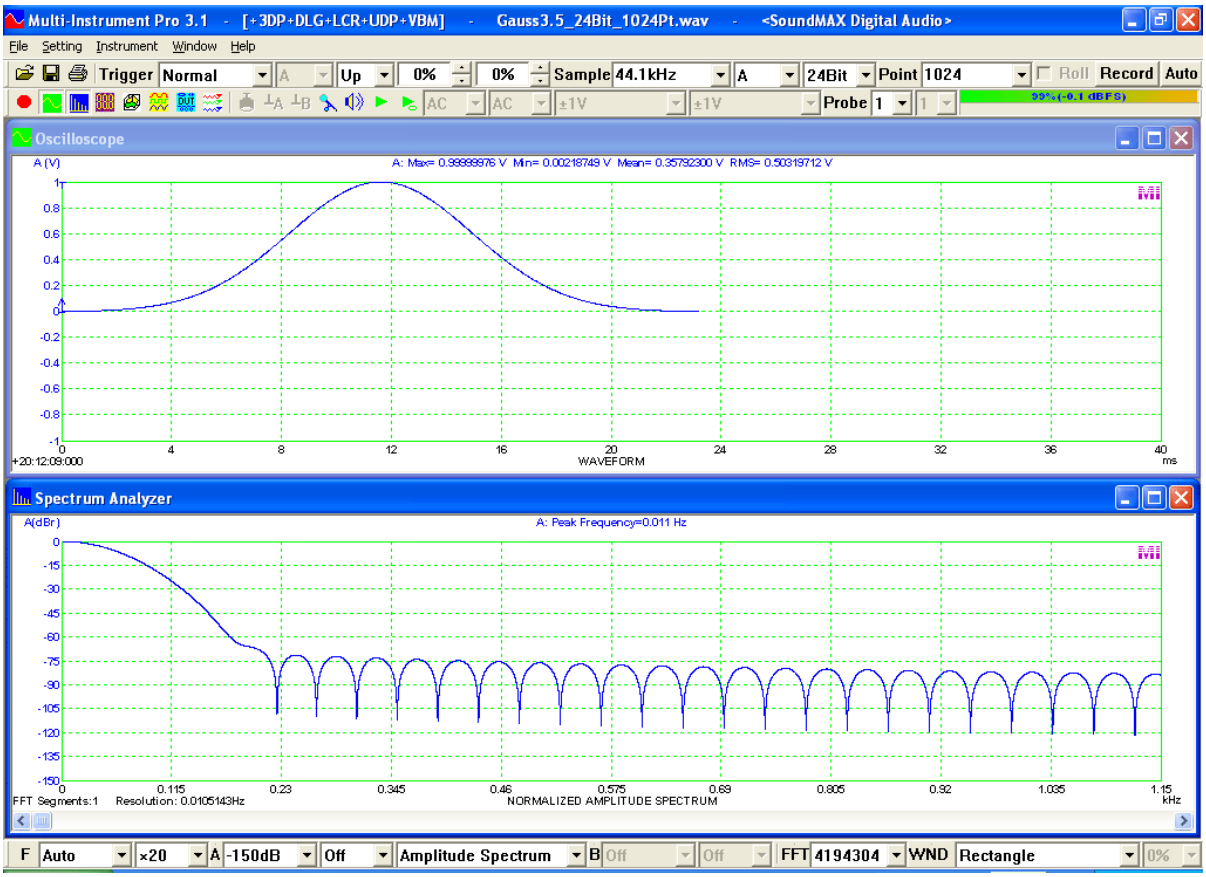
3.13 Gaussian Window ($\alpha = 3.5$)

$w(n) = e^{-0.5[\alpha\,(2n/N-1)]\,[\alpha\,(2n/N-1)]}$, $\alpha >= 2$, $n = 0,1,\ldots,N-1$;

$\alpha = 3.5$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-71	-6	1.85	2.62	0.87	0.36	1.98



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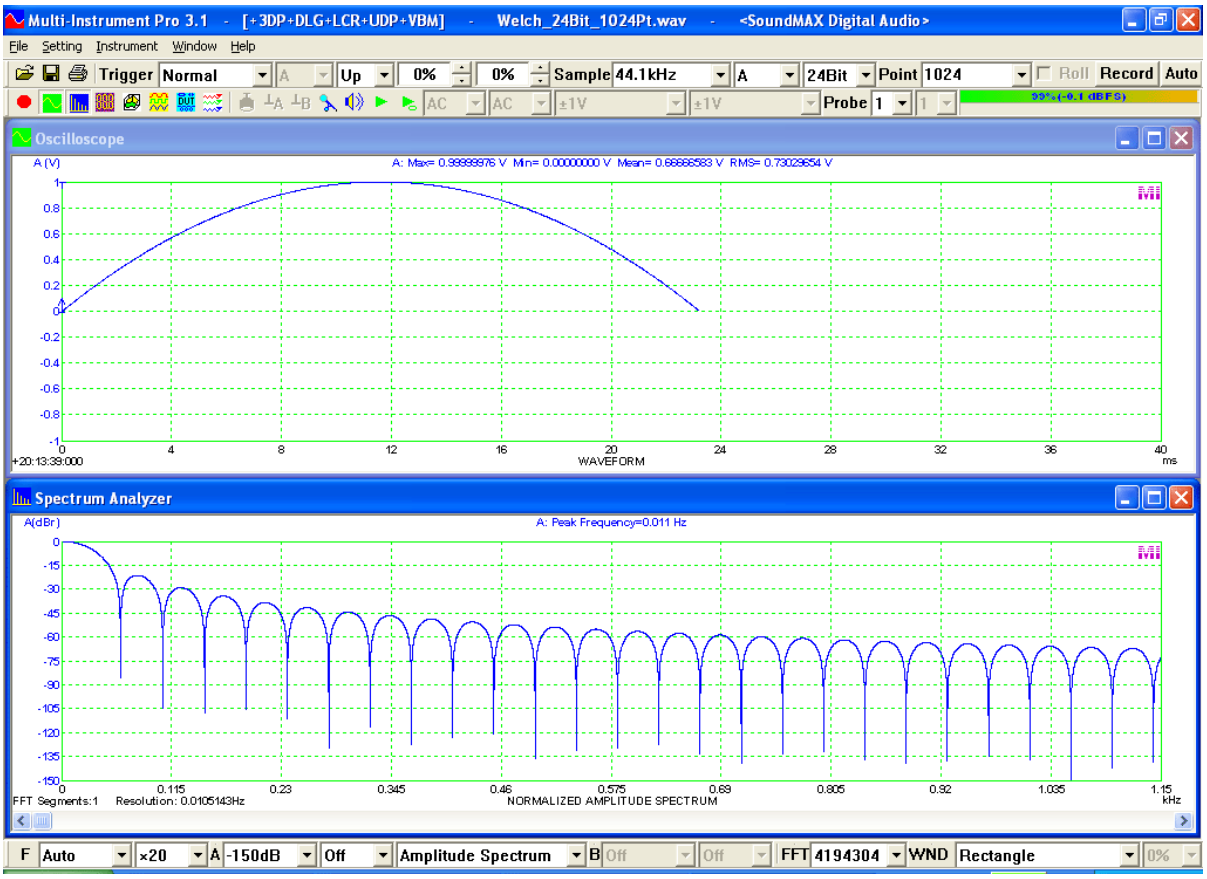
3.14 Welch (Riesz) Window

$w(n) = 1 - [2n/N-1]^2, \quad n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-21	-12	1.15	1.59	2.22	0.67	1.20



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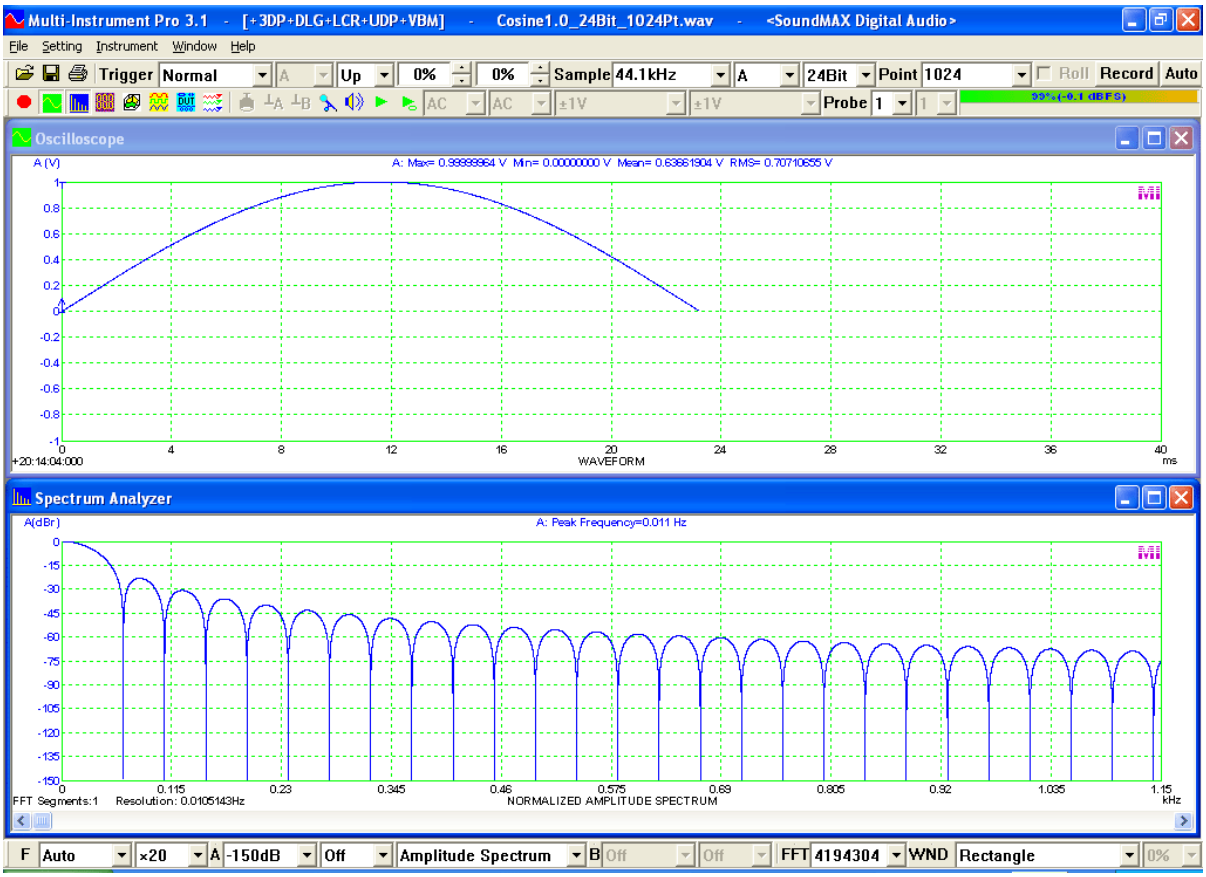
3.15 Cosine Window (α = 1)

$w(n) = \sin^{\alpha}(n\pi/N)$, $n = 0,1,\ldots,N-1$;

$\alpha = 1$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-23	-12	1.19	1.64	2.10	0.64	1.23



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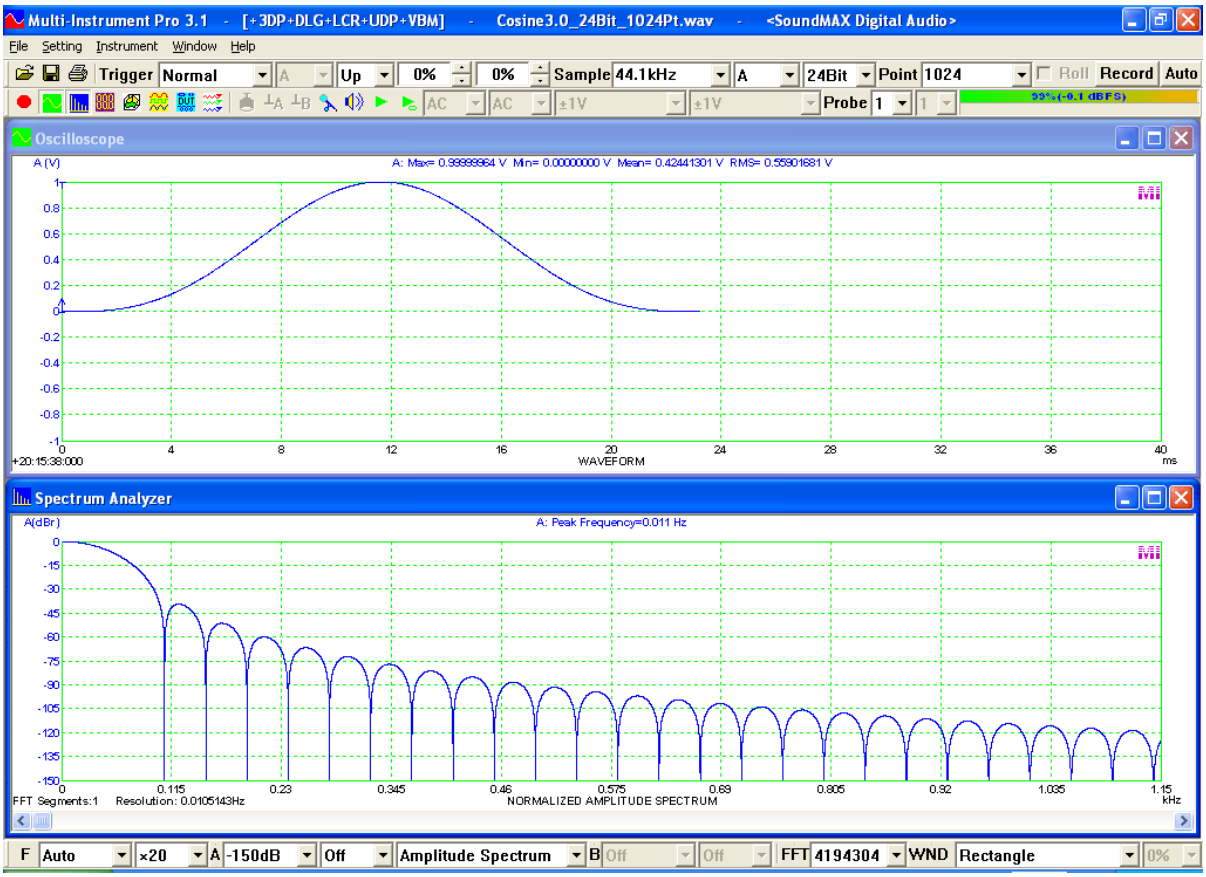
3.16 Cosine Window (α = 3)

$w(n) = \sin^{\alpha}(n\pi/N)$, $n = 0,1,\ldots,N-1$;

$\alpha = 3$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-39	-24	1.66	2.31	1.08	0.42	1.73



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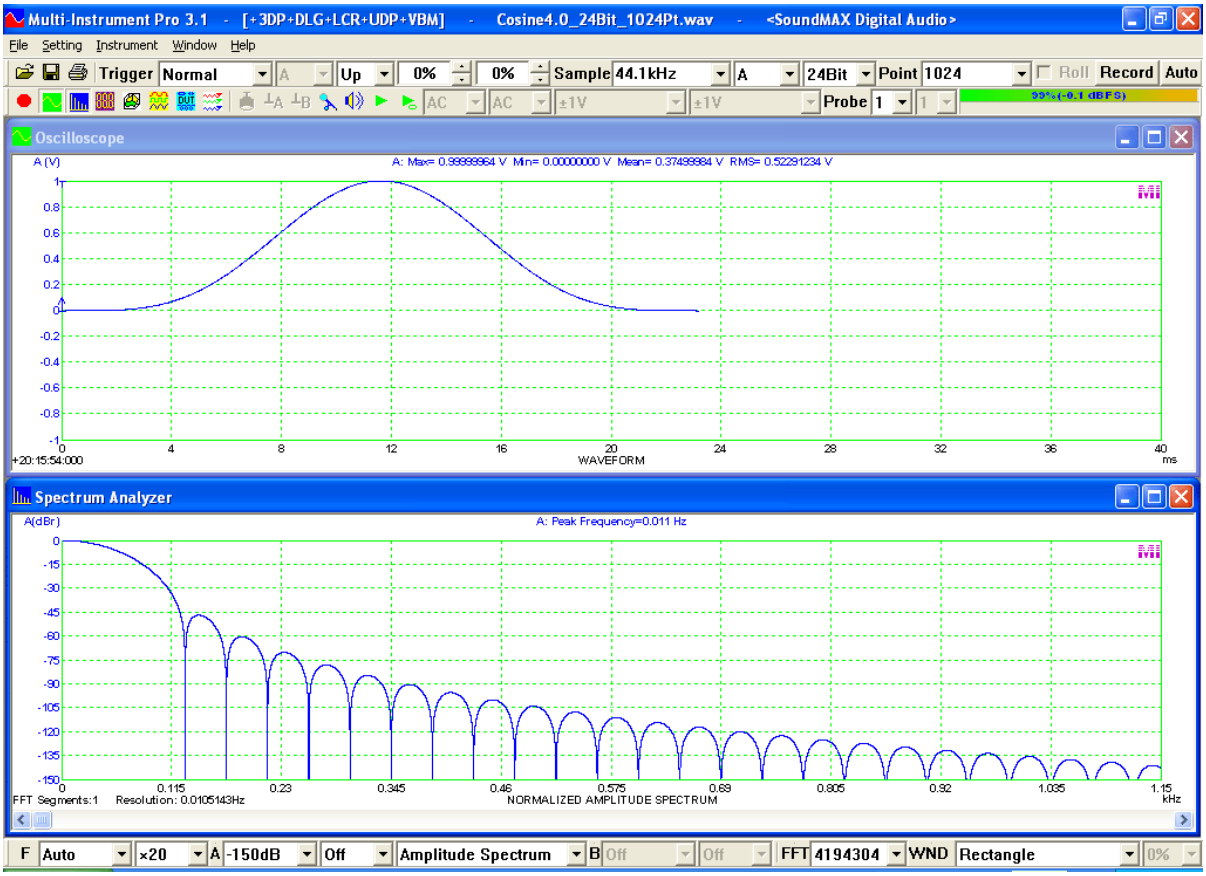
3.17 Cosine Window ($\alpha = 4$)

$w(n) = \sin^{\alpha}(n\pi/N), \quad n = 0,1,\ldots,N-1;$

$\alpha = 4$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-47	-30	1.85	2.58	0.86	0.38	1.94



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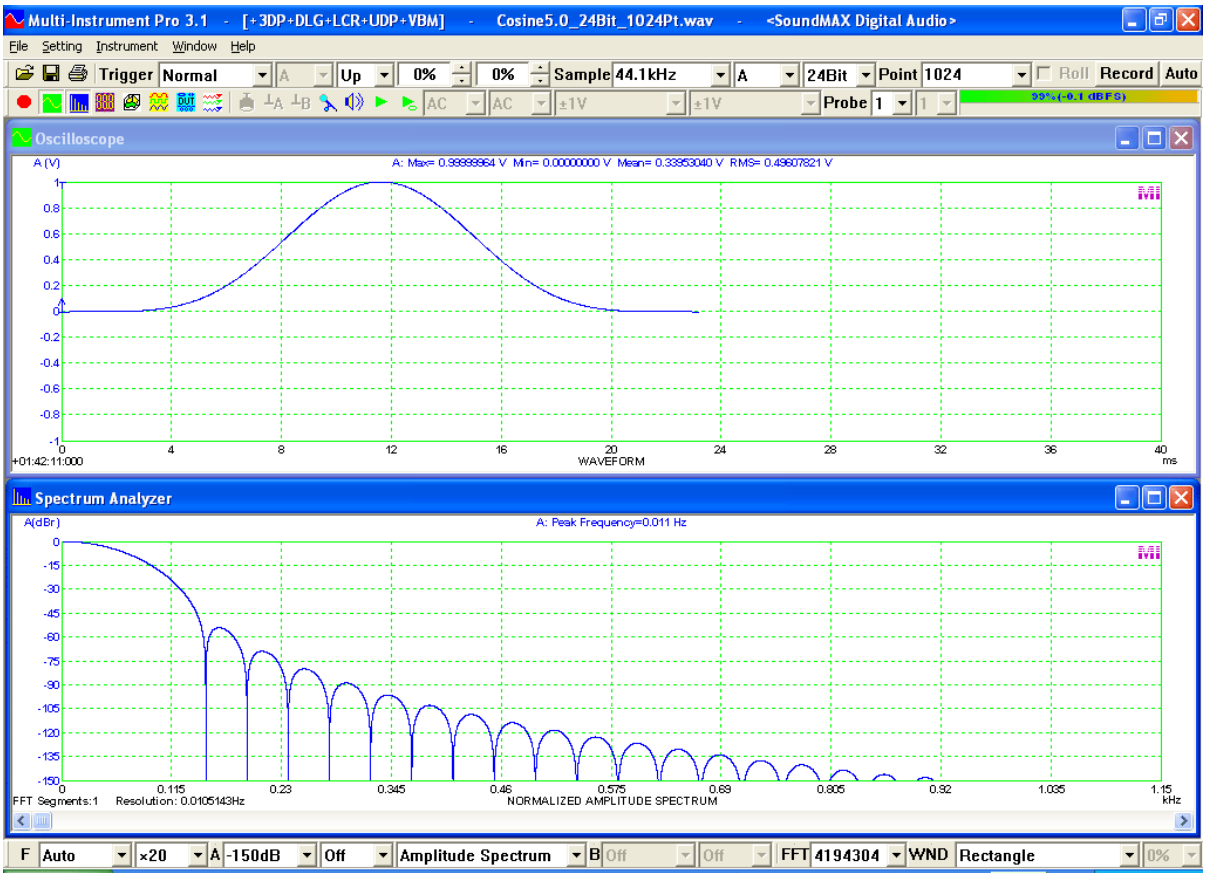
3.18 Cosine Window (α = 5)

$w(n) = \sin^{\alpha}(n\pi/N), \quad n = 0,1,\ldots,N-1;$

$\alpha = 5$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-54	-36	2.03	2.84	0.72	0.34	2.13



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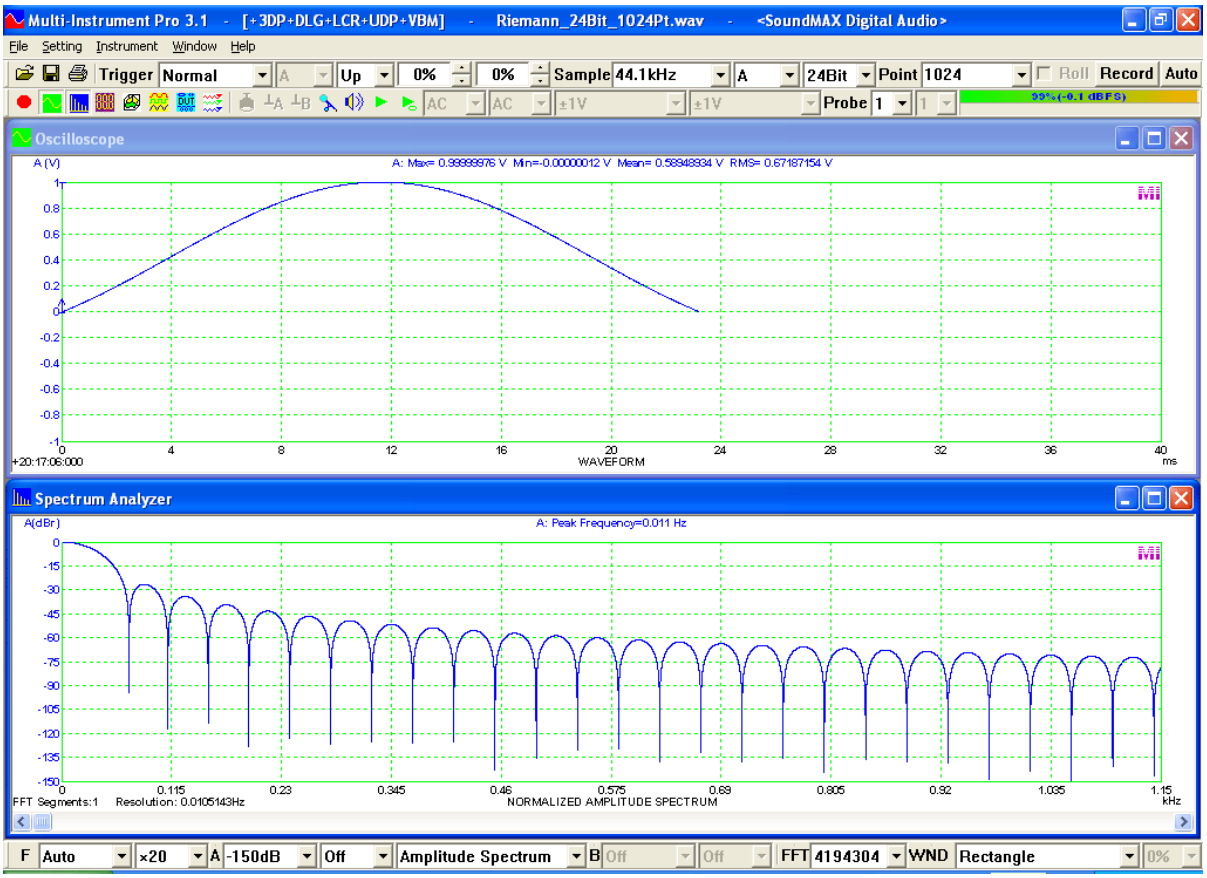
3.19 Riemann Window

$w(n) = \sin[(2n/N-1)\,\pi]/[(2n/N-1)\,\pi], \quad n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-26	-12	1.25	1.73	1.89	0.59	1.30



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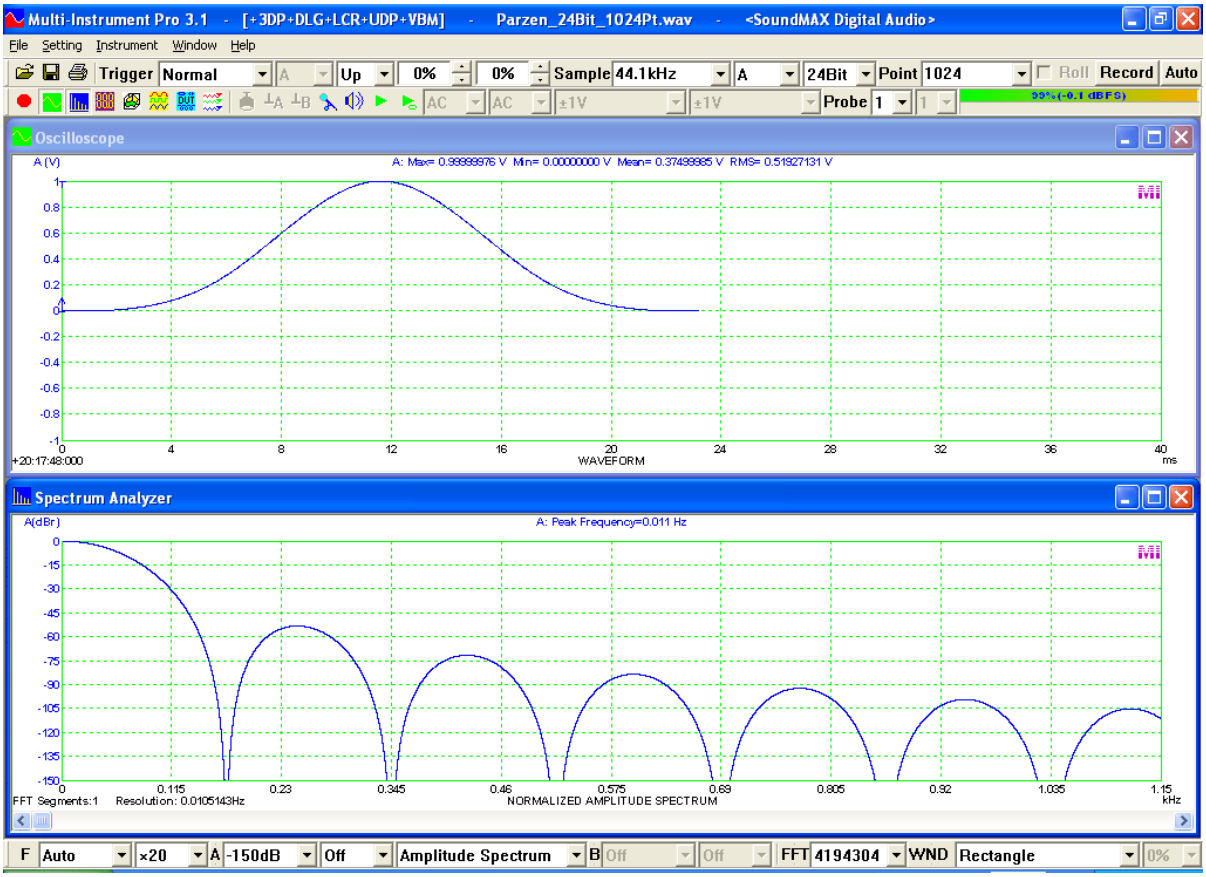
3.20 Parzen (De La Valle-Poussin) Window

$w(n) = 1 - 6(2n/N-1)^2(1-|2n/N-1|)$ $\quad 0 <= |n-N/2| <= (N/4)$

$w(n) = 2(1-|2n/N-1|)^3$ $\quad (N/4) <= |n-N/2| <= (N/2)$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-53	-24	1.82	2.55	0.90	0.38	1.92



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3.21 Tukey (Tapered Cosine) Window (α = 0.25)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$

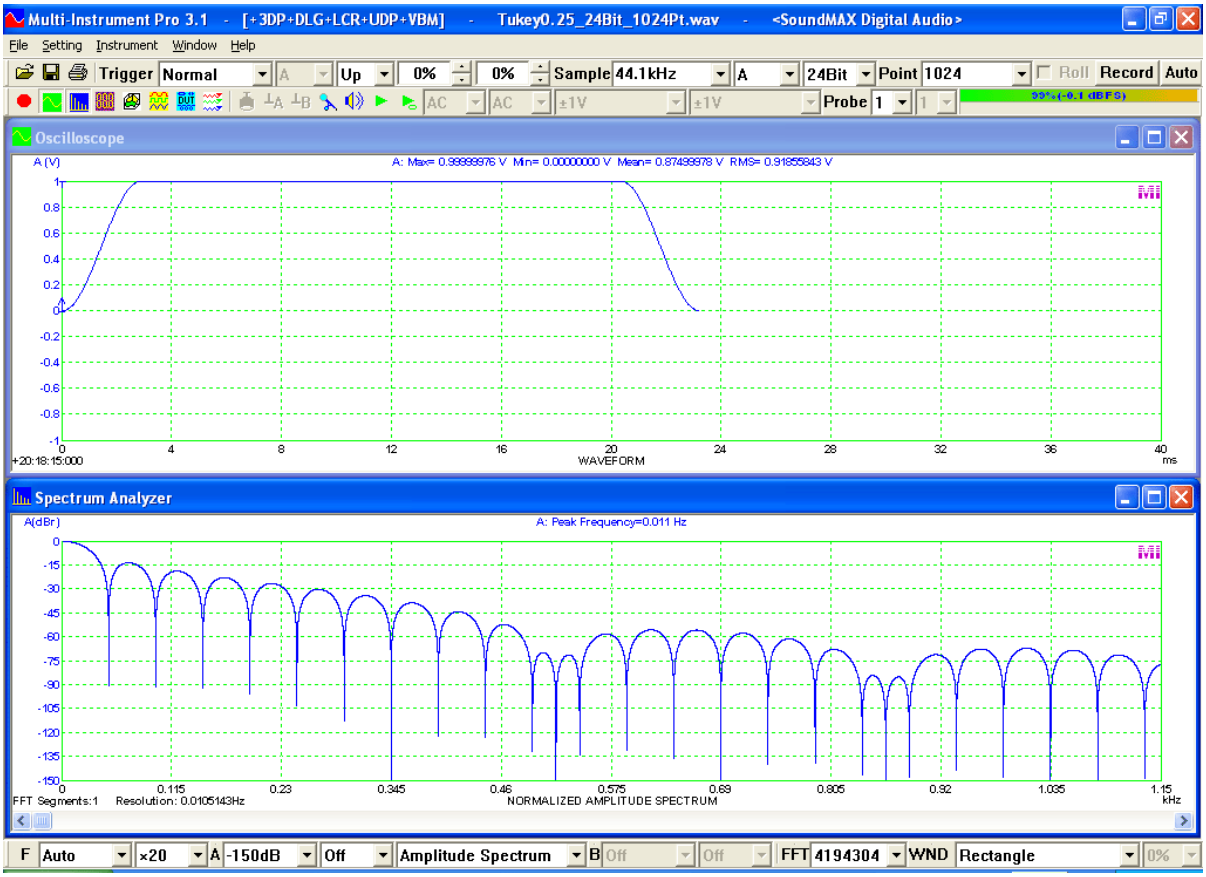
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$

$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

$\alpha = 0.25$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-14	-18	1.01	1.37	2.97	0.88	1.10



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3.22 Tukey (Tapered Cosine) Window (α = 0.50)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$

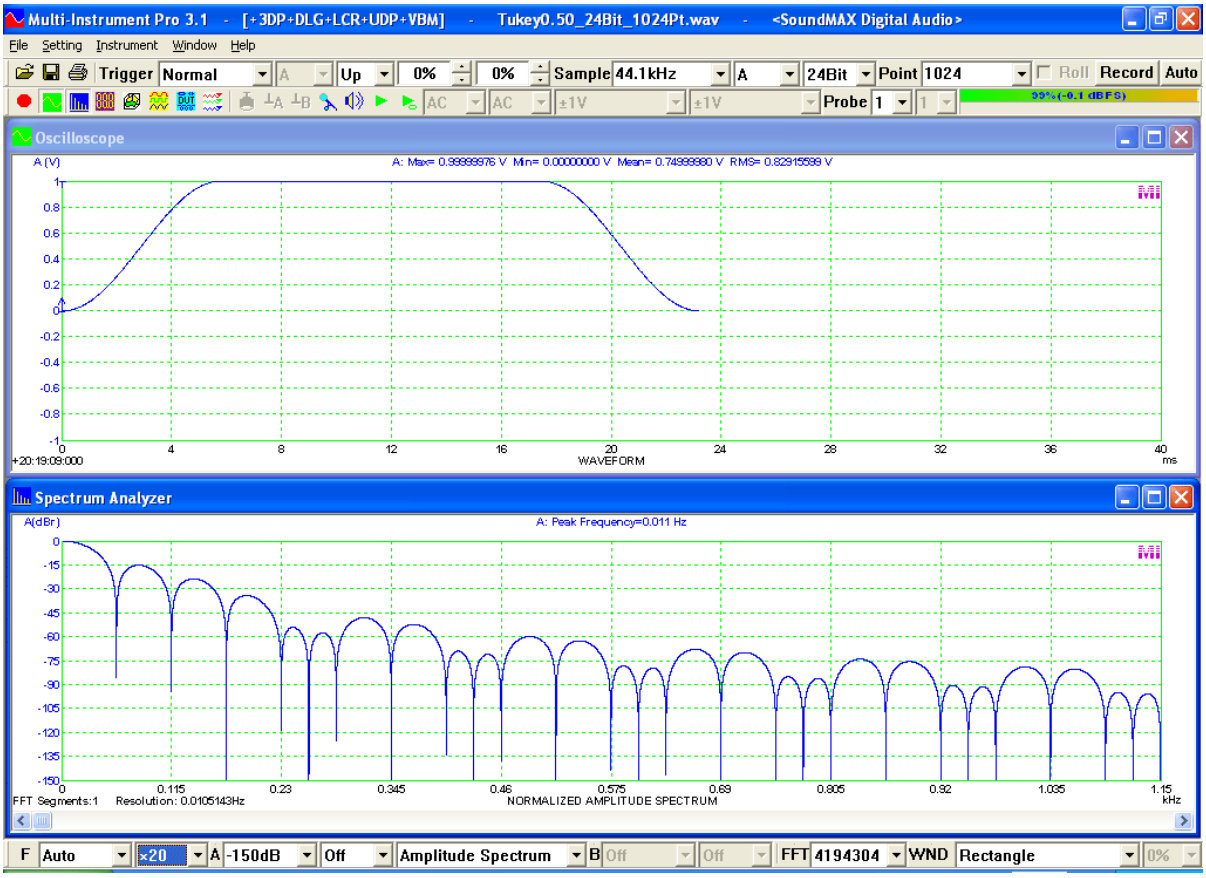
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$

$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

$\alpha = 0.50$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-15	-18	1.15	1.57	2.24	0.75	1.22



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3.23 Tukey (Tapered Cosine) Window (α = 0.75)

$w(n) = 0.5\{1 - \cos[2\pi n/(\alpha N)]\}$ $\quad n < (\alpha N/2)$

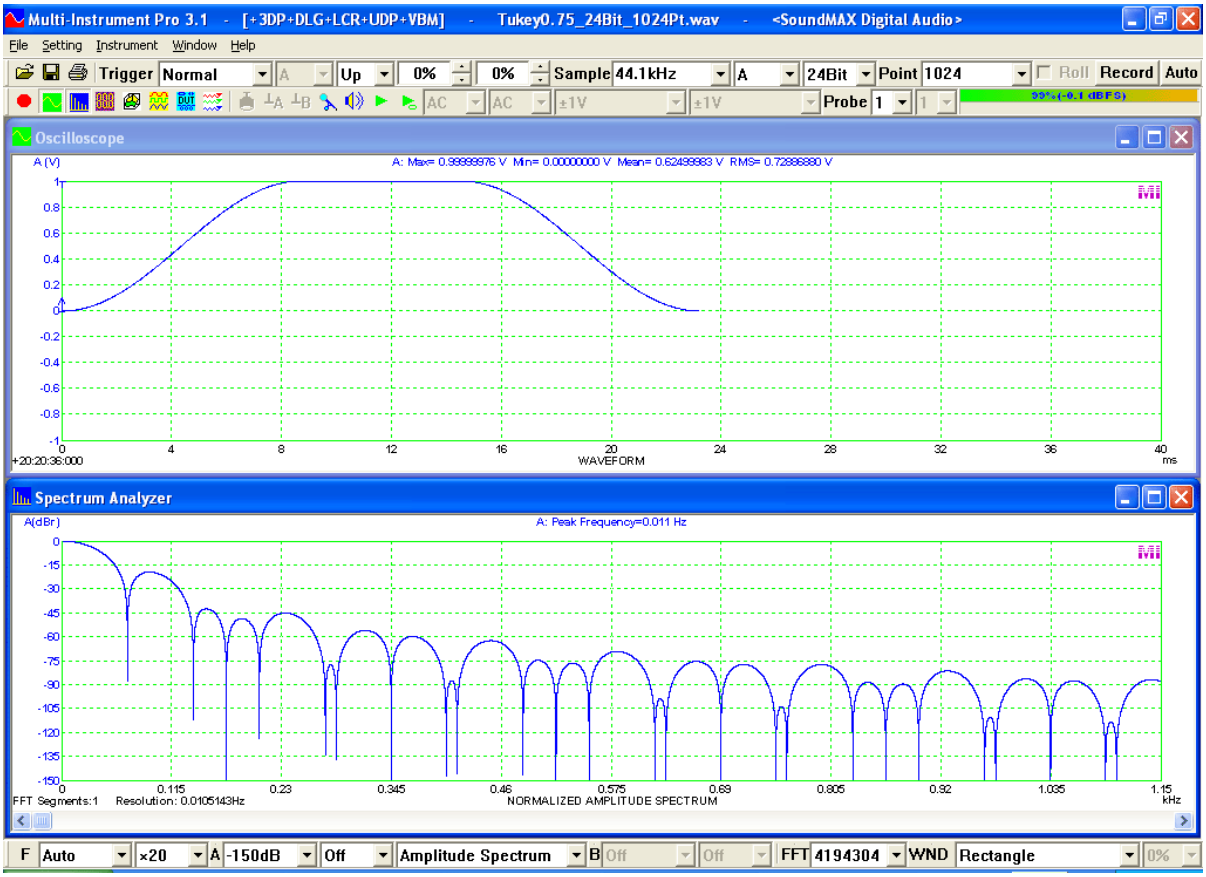
$w(n) = 1.0$ $\quad (\alpha N/2) <= n <= N-(\alpha N/2)$

$w(n) = 0.5\{1 - \cos[2\pi/\alpha - 2\pi n/(\alpha N)]\}$ $\quad n > N-(\alpha N/2)$

$n = 0,1,\ldots,N-1;$

$\alpha = 0.75$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-19	-18	1.30	1.79	1.73	0.63	1.36



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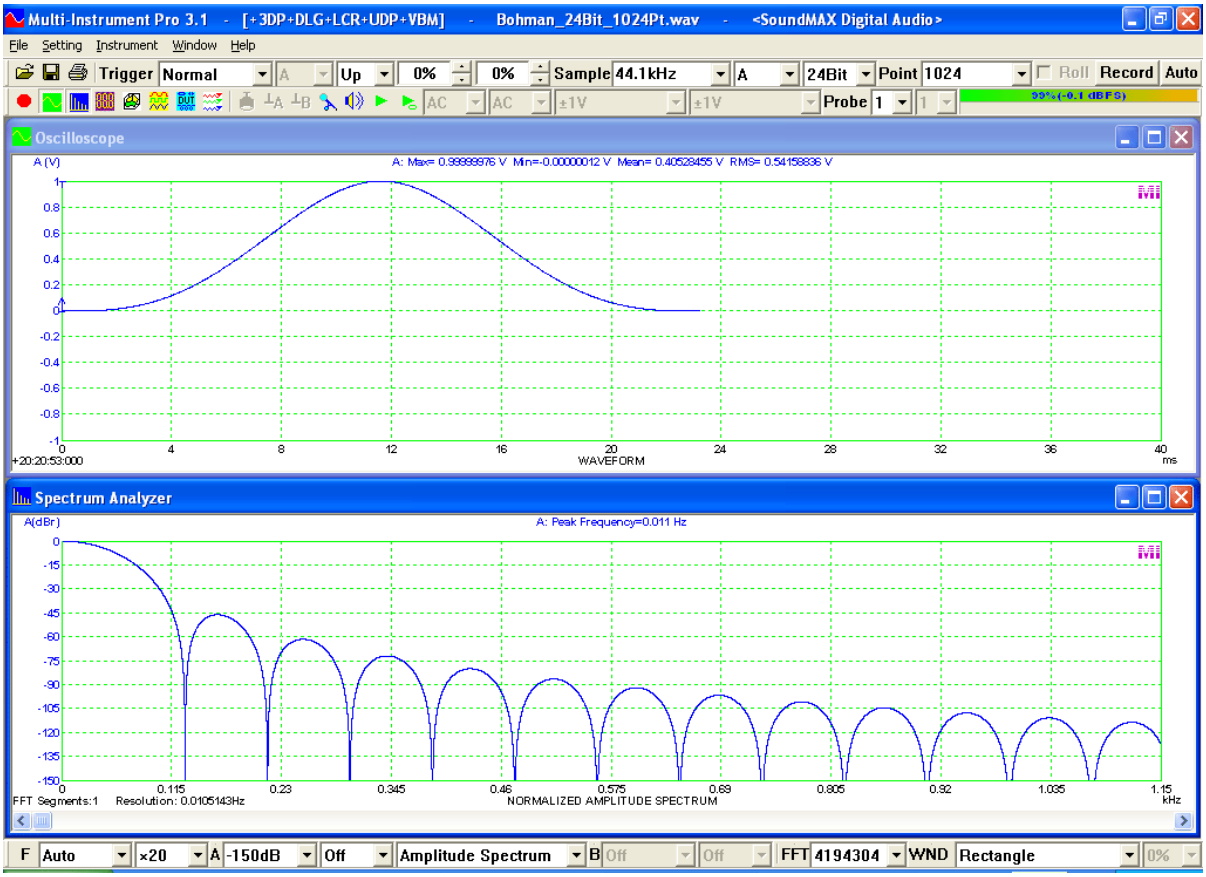
3.24 Bohman Window

$$w(n) = (1 - |2n/N - 1|)\cos(\pi|2n/N - 1|) + \sin(\pi|2n/N - 1|)/\pi, \quad n = 0,1,\ldots,N-1;$$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-46	-24	1.70	2.37	1.02	0.41	1.79



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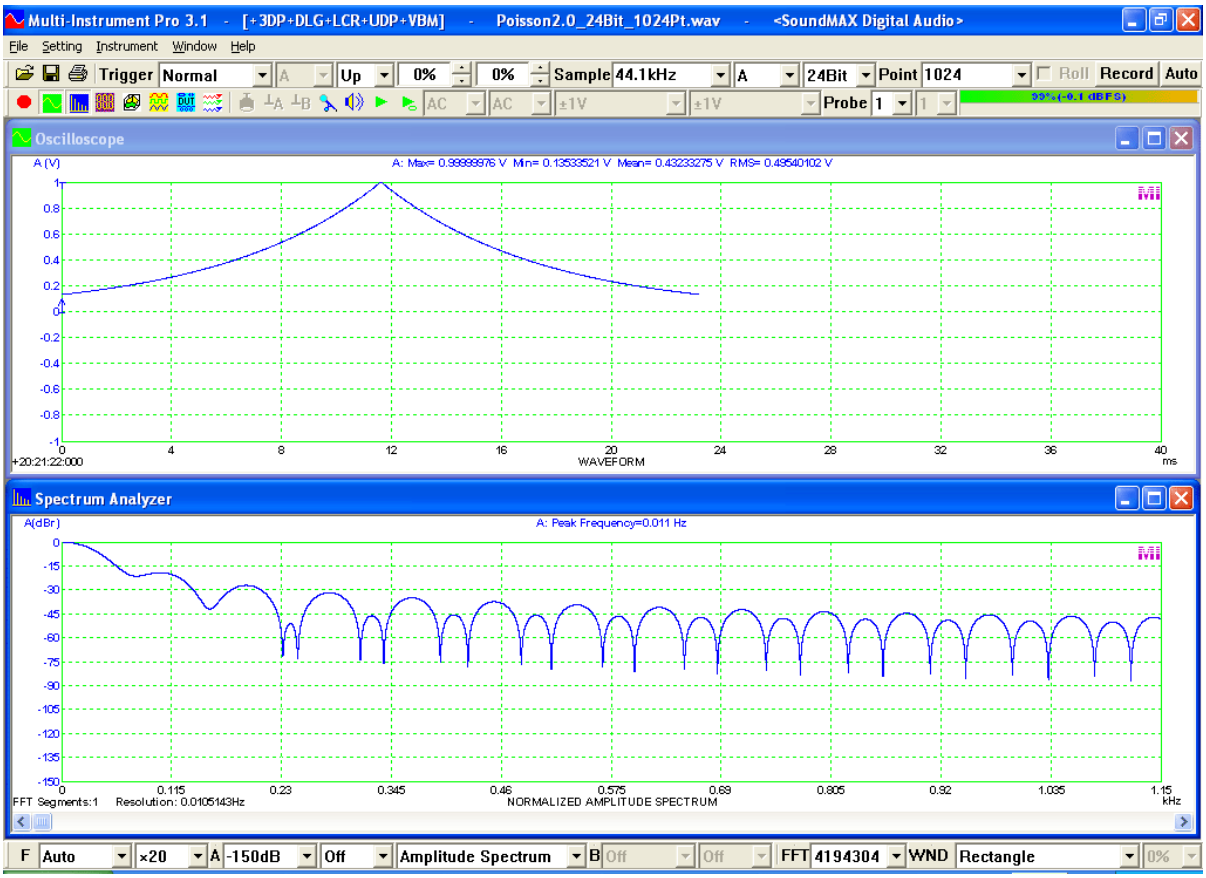
3.25 Poisson Window (α = 2)

$w(n) = e^{(-\alpha|2n/N-1|)}$, $n = 0,1,\ldots,N-1;$

α = 2

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-19	-6	1.21	1.70	2.03	0.43	1.31



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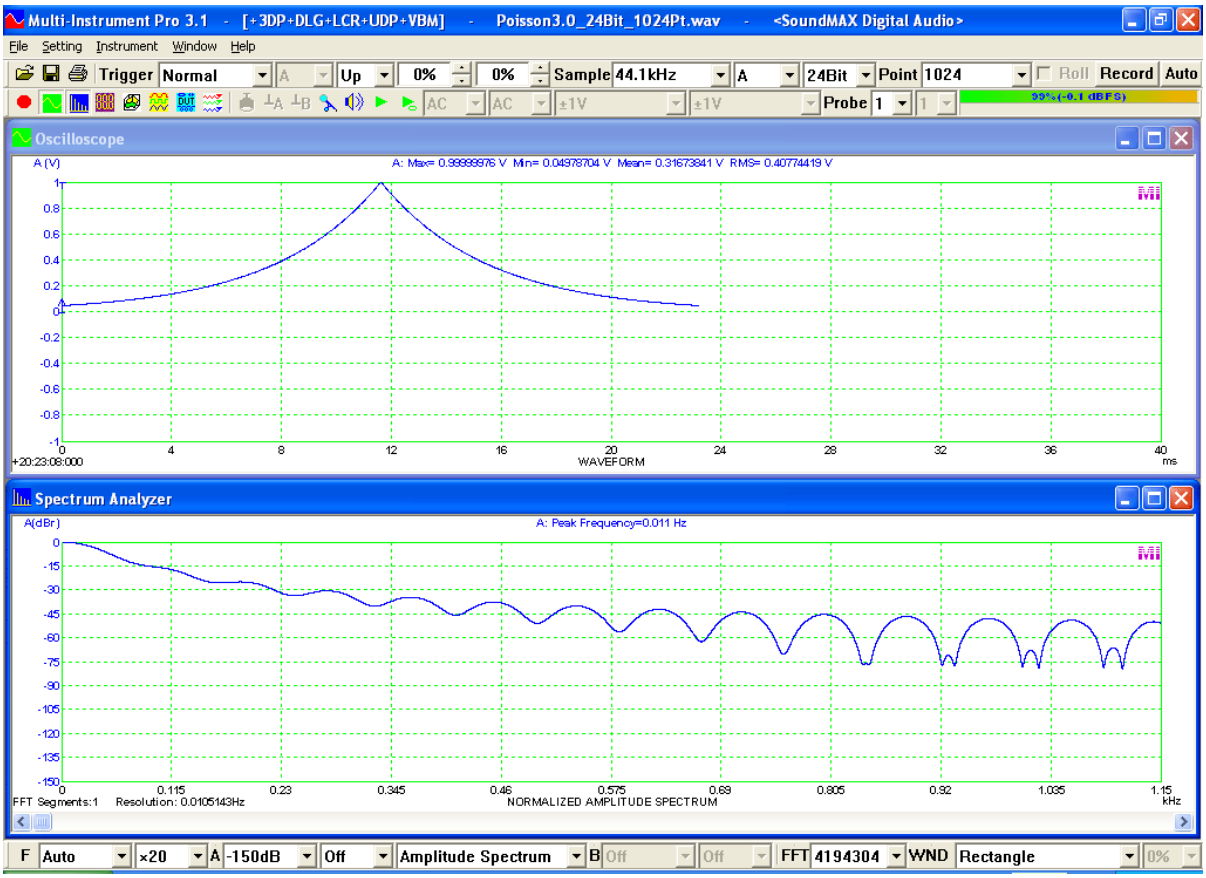
3.26 Poisson Window (α = 3)

$w(n) = e^{(-\alpha|2n/N-1|)}$, n = 0,1,…,N-1;

α = 3

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-25	-6	1.45	2.08	1.44	0.32	1.66



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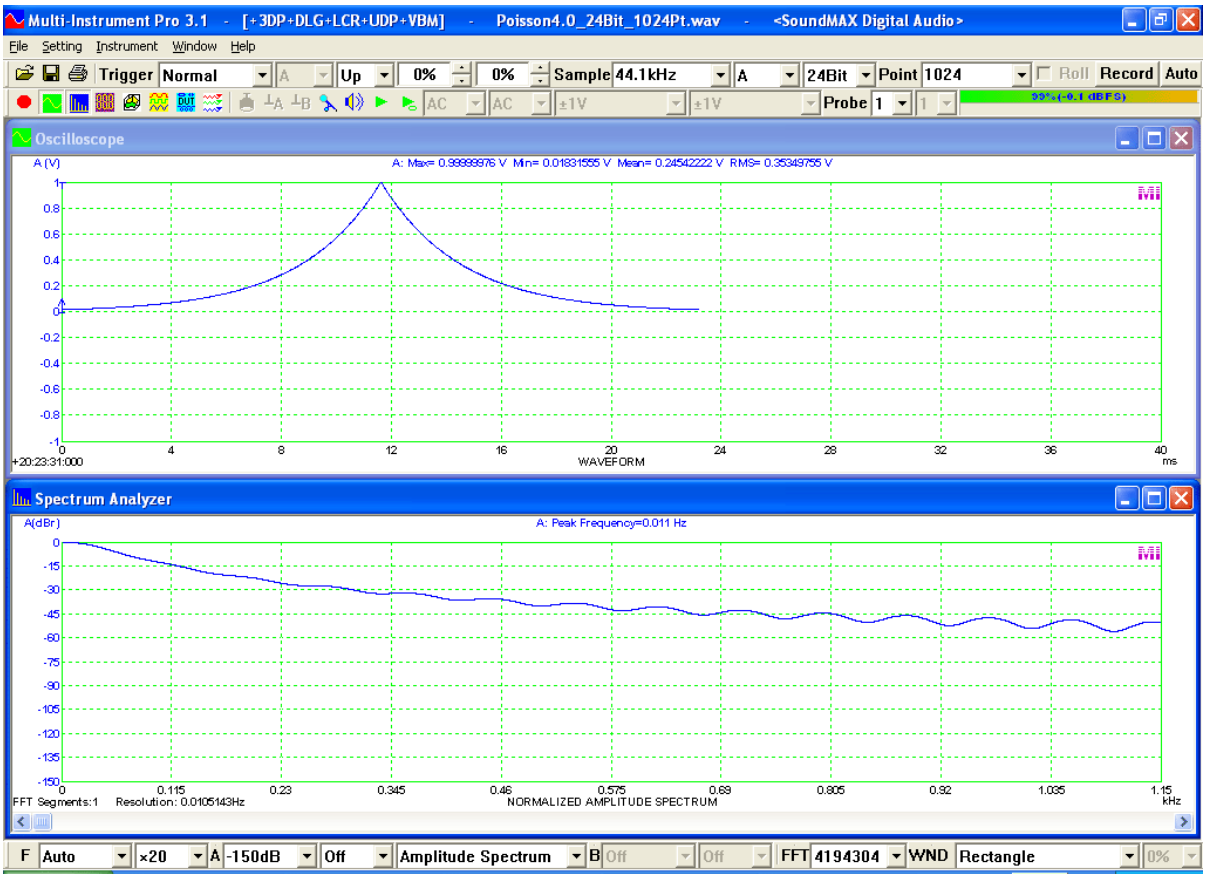
3.27 Poisson Window ($\alpha = 4$)

$w(n) = e^{(-\alpha|2n/N-1|)}, \quad n = 0,1,\ldots,N-1;$

$\alpha = 4$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-31	-6	1.75	2.58	1.02	0.25	2.07



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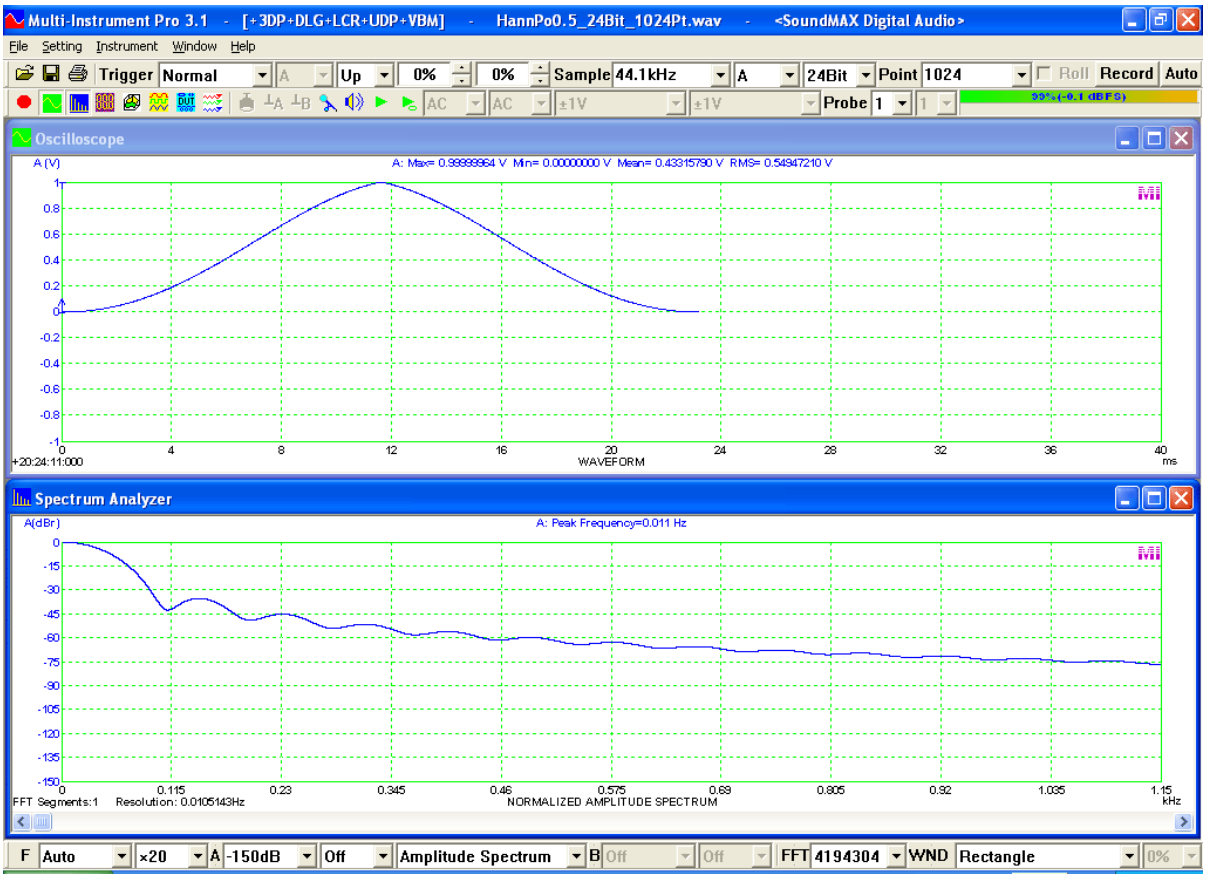
3.28 Hanning-Poisson Window ($\alpha = 0.5$)

$w(n) = [0.5-0.5\cos(2n\pi/N)]e^{(-\alpha|2n/N-1|)}$, $n = 0,1,\ldots,N-1;$

$\alpha = 0.5$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-35	-12	1.53	2.14	1.26	0.43	1.61



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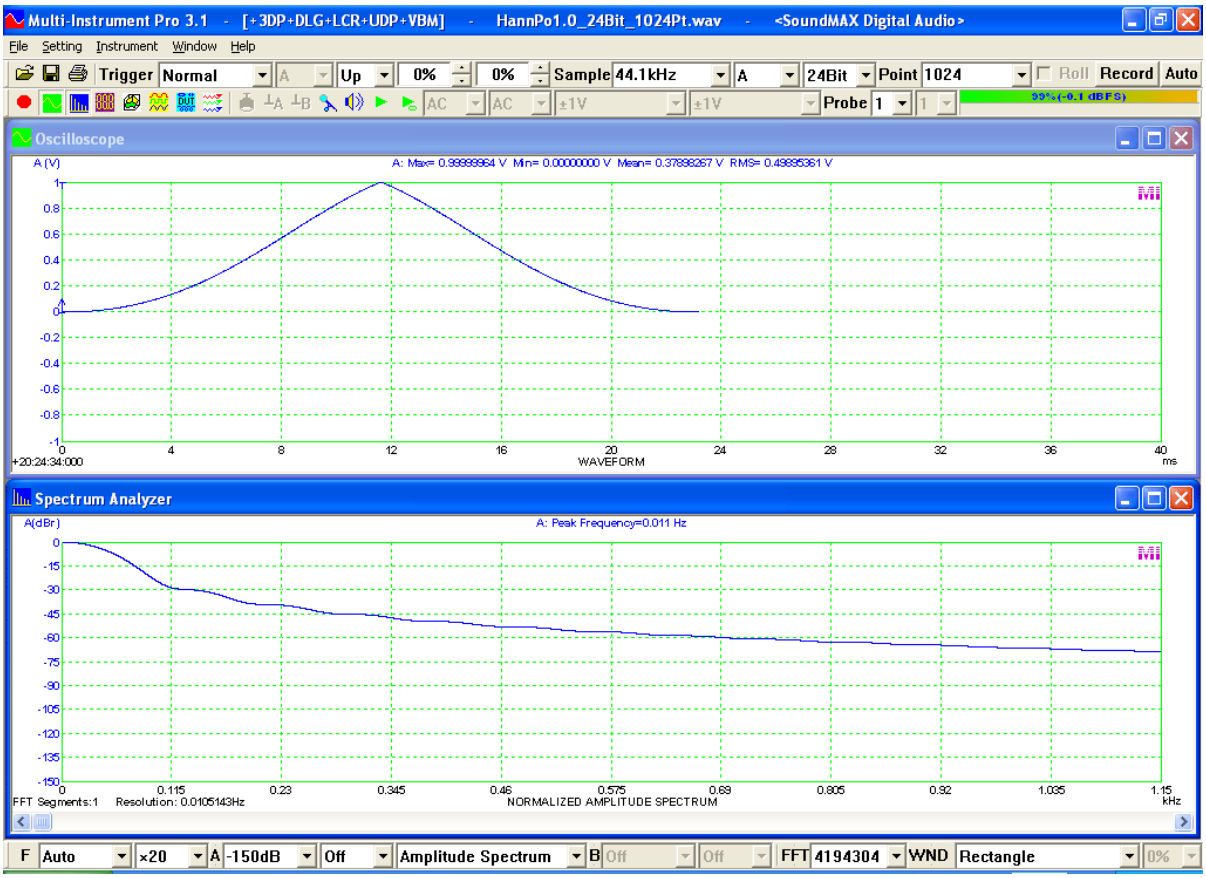
3.29 Hanning-Poisson Window (α = 1.0)

$w(n) = [0.5-0.5\cos(2n\pi/N)]e^{(-\alpha|2n/N-1|)}$, n = 0,1,…,N-1;

α = 1.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-39	-12	1.63	2.29	1.11	0.38	1.73



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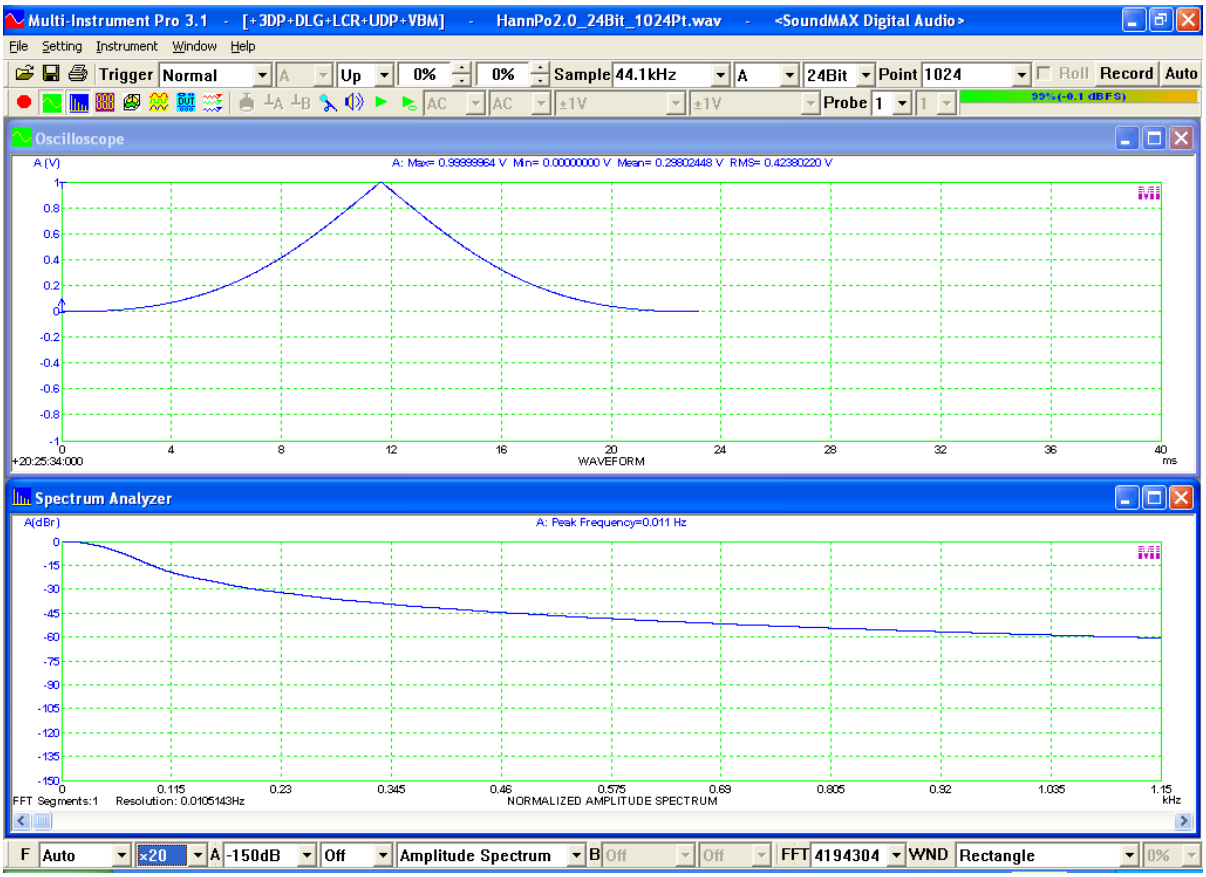
3.30 Hanning-Poisson Window (α = 2.0)

$w(n) = [0.5-0.5\cos(2n\pi/N)]e^{(-\alpha|2n/N-1|)}$, $n = 0,1,\ldots,N-1;$

$\alpha = 2.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
N.A	-12	1.86	2.64	0.87	0.30	2.02



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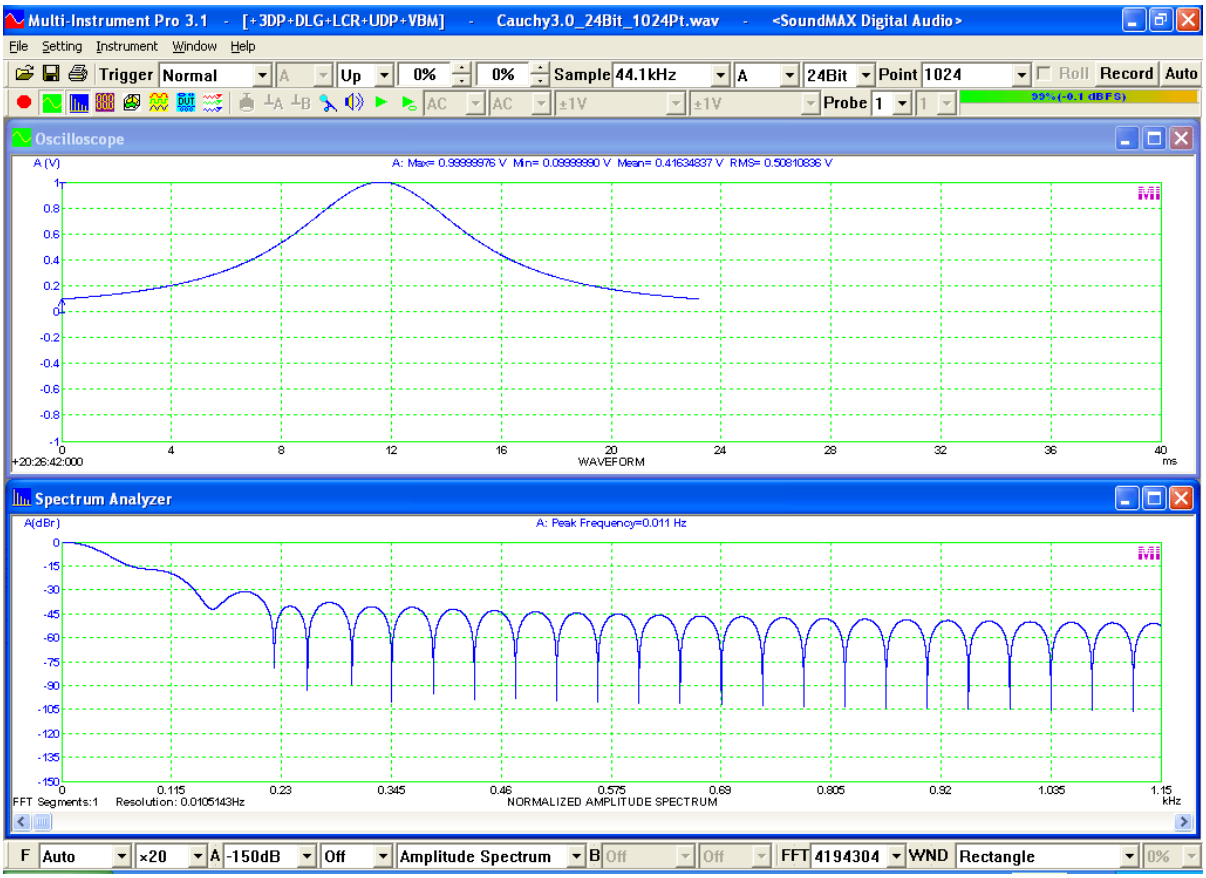
3.31 Cauchy Window ($\alpha = 3.0$)

$w(n) = [1 + \alpha(2n/N - 1)^2]^{-1}, \quad n = 0,1,\ldots,N-1;$

$\alpha = 3.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-31	-6	1.34	1.91	1.67	0.42	1.49



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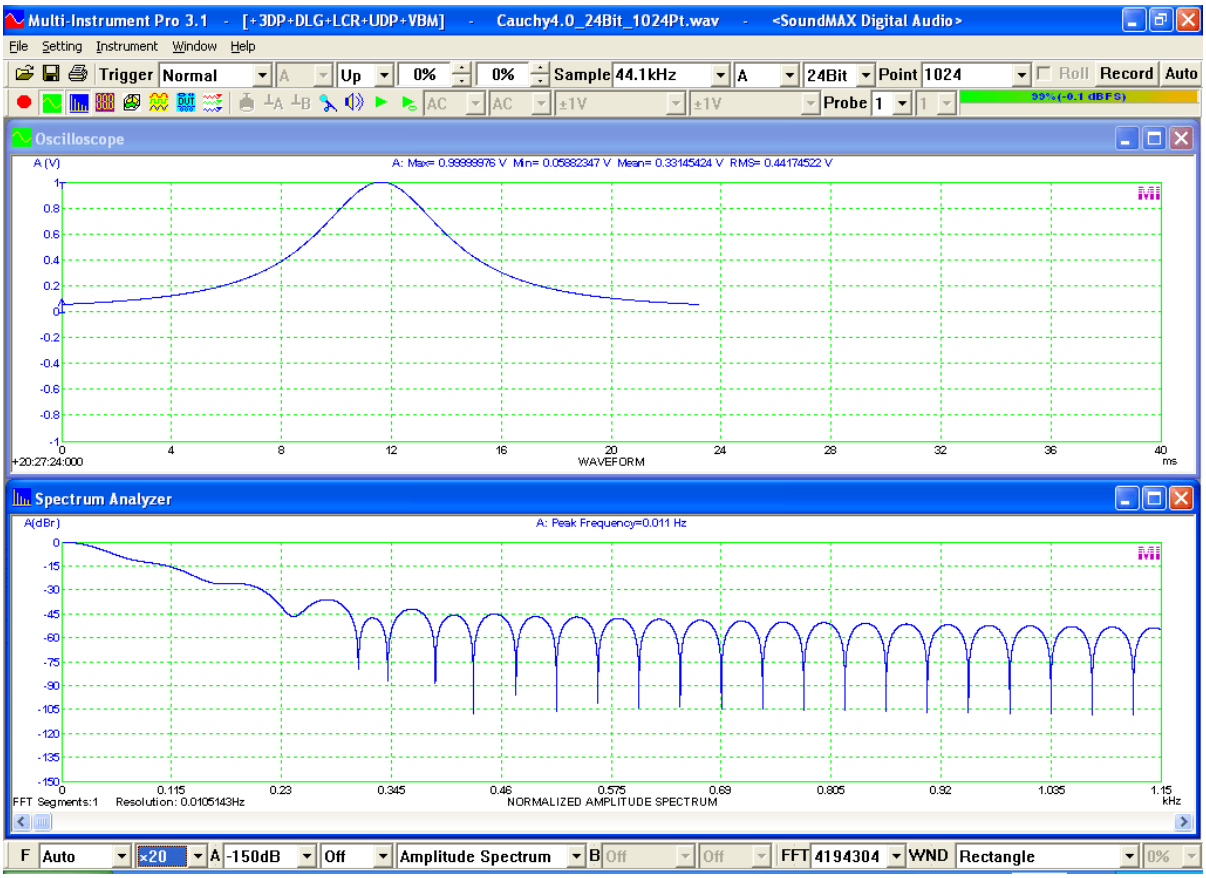
3.32 Cauchy Window ($\alpha = 4.0$)

$w(n) = [1 + \alpha(2n/N - 1)^2]^{-1}, \quad n = 0,1,\ldots,N-1;$

$\alpha = 4.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-36	-6	1.52	2.21	1.33	0.33	1.78



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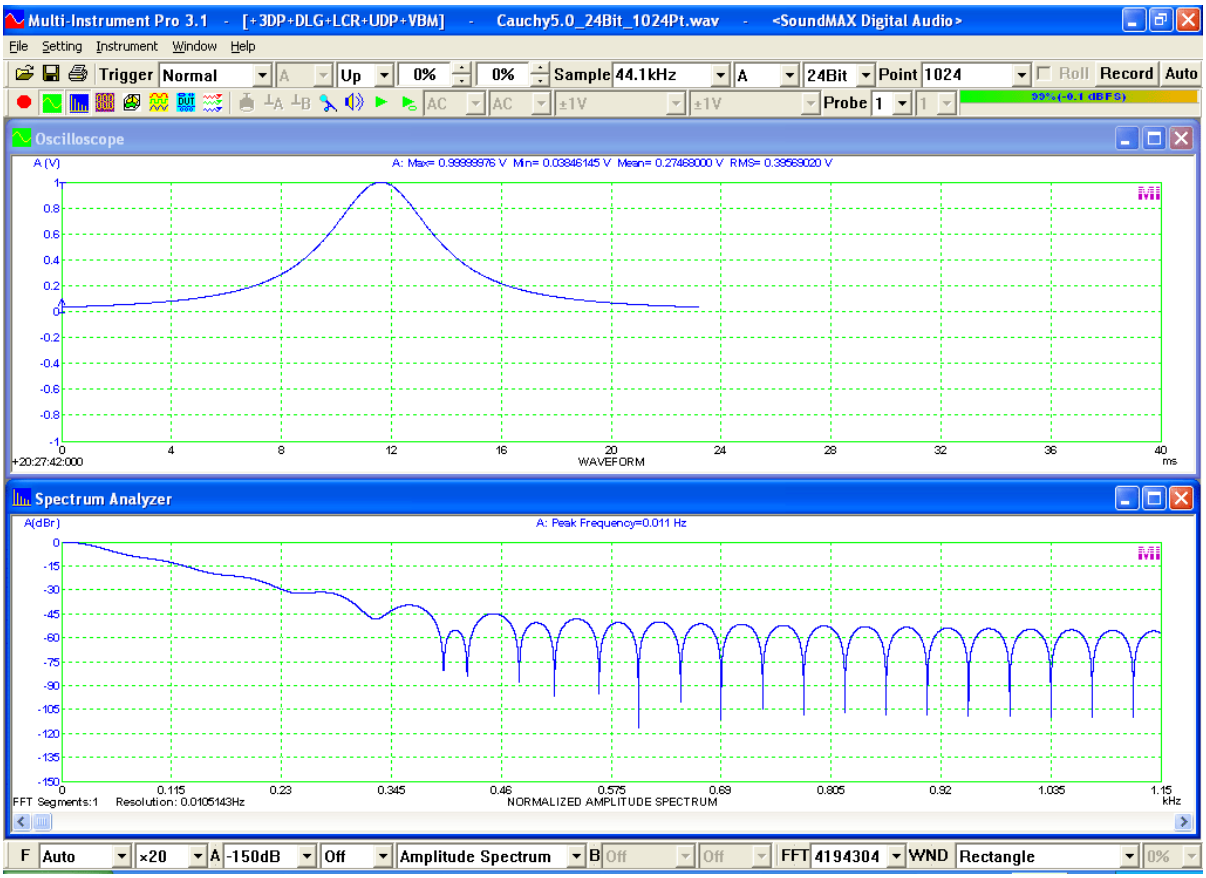
3.33 Cauchy Window ($\alpha = 5.0$)

$w(n) = [1 + \alpha(2n/N - 1)^2]^{-1}, \quad n = 0,1,\ldots,N-1;$

$\alpha = 5.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-39	-6	1.69	2.54	1.11	0.27	2.07



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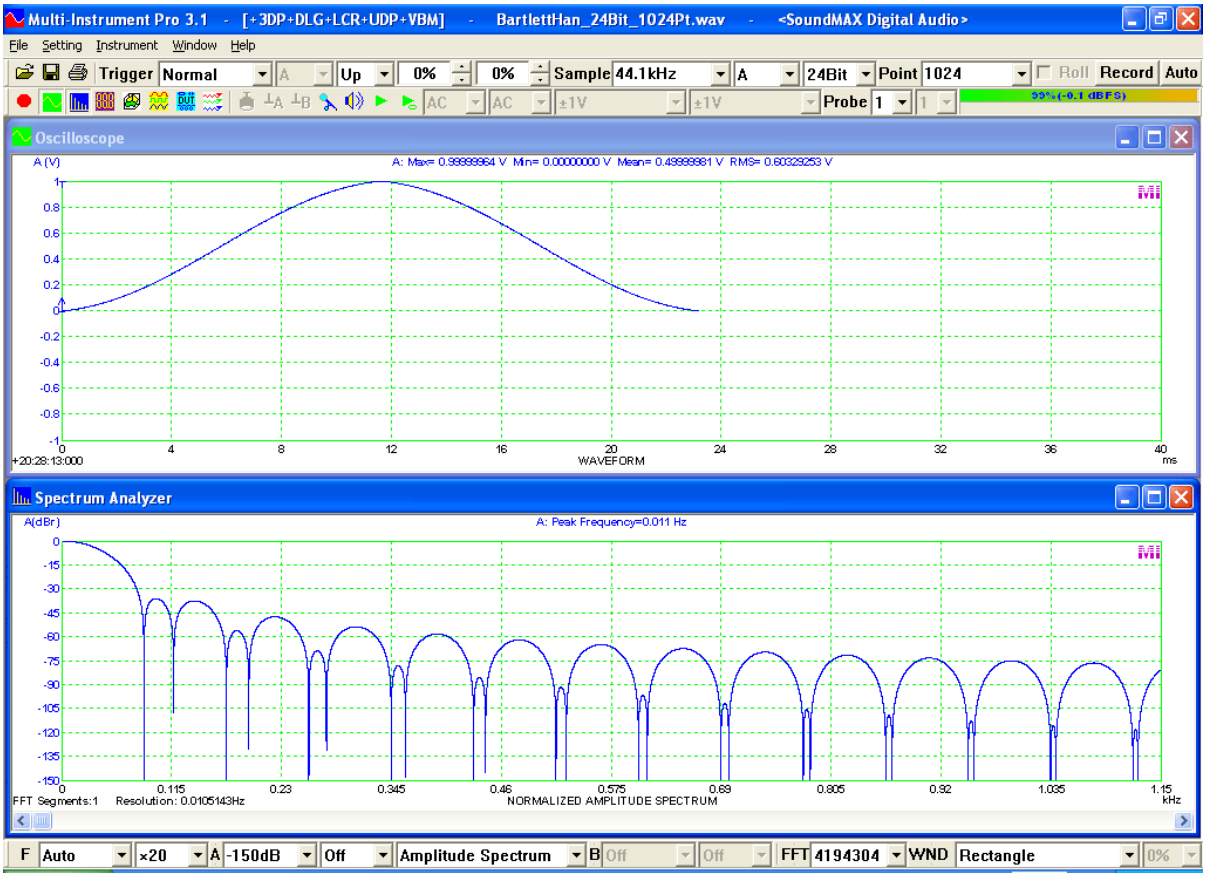
3.34 Bartlett-Hann Window

$w(n) = 0.62-0.48|n/N-0.5|-0.38\cos(2n\pi/N), \quad n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-36	-12	1.40	1.94	1.52	0.50	1.46



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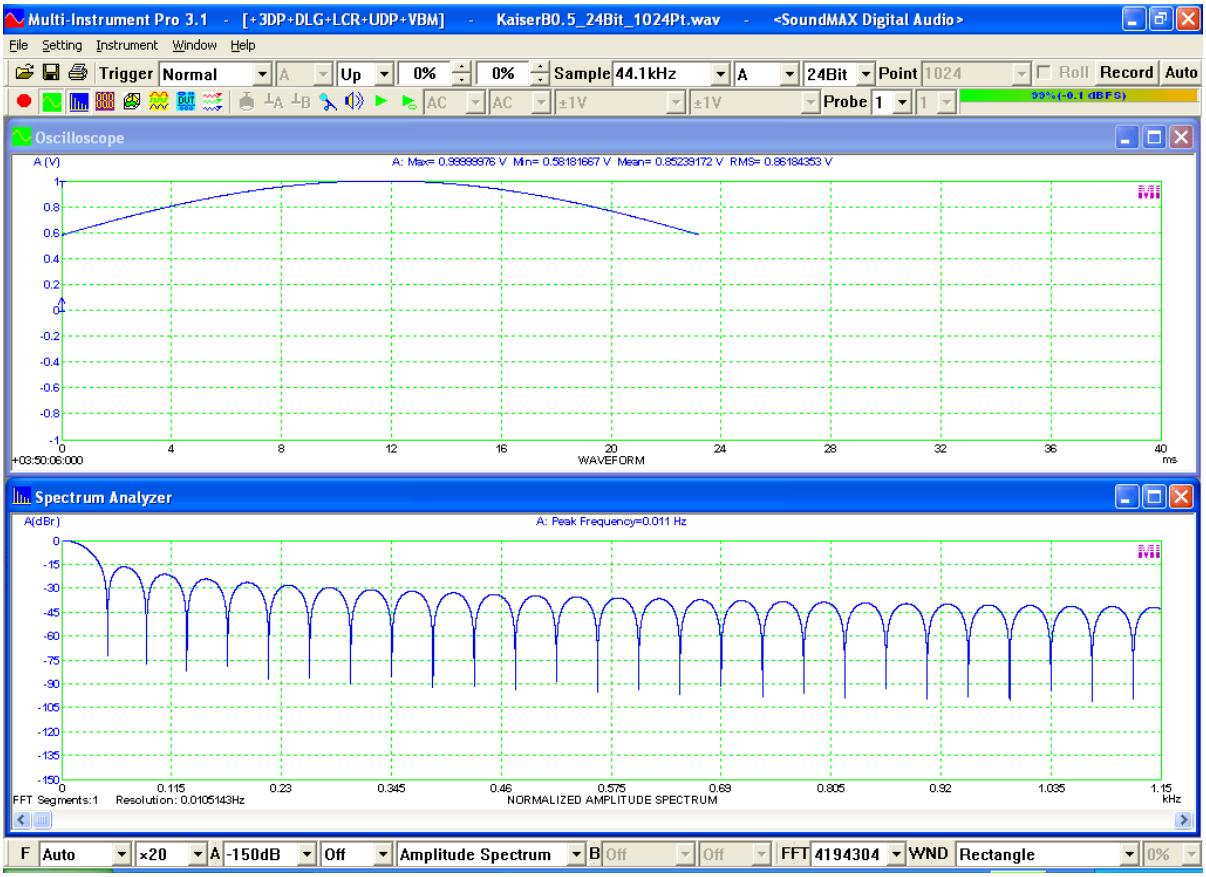
3.35 Kaiser-Bessel Window ($\alpha = 0.5$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 0.5$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-17	-6	0.95	1.31	3.32	0.85	1.02



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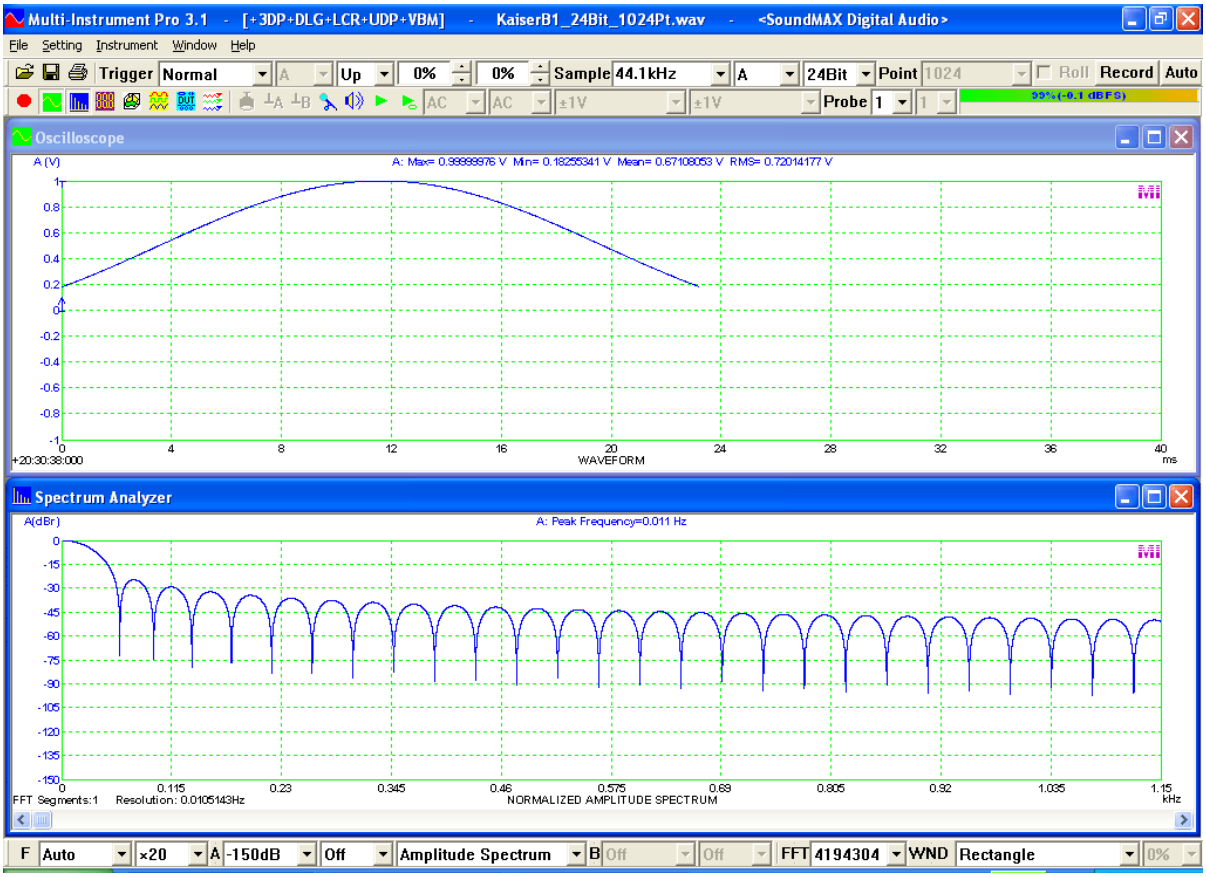
3.36 Kaiser-Bessel Window ($\alpha = 1.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 1.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-25	-6	1.11	1.53	2.43	0.67	1.15



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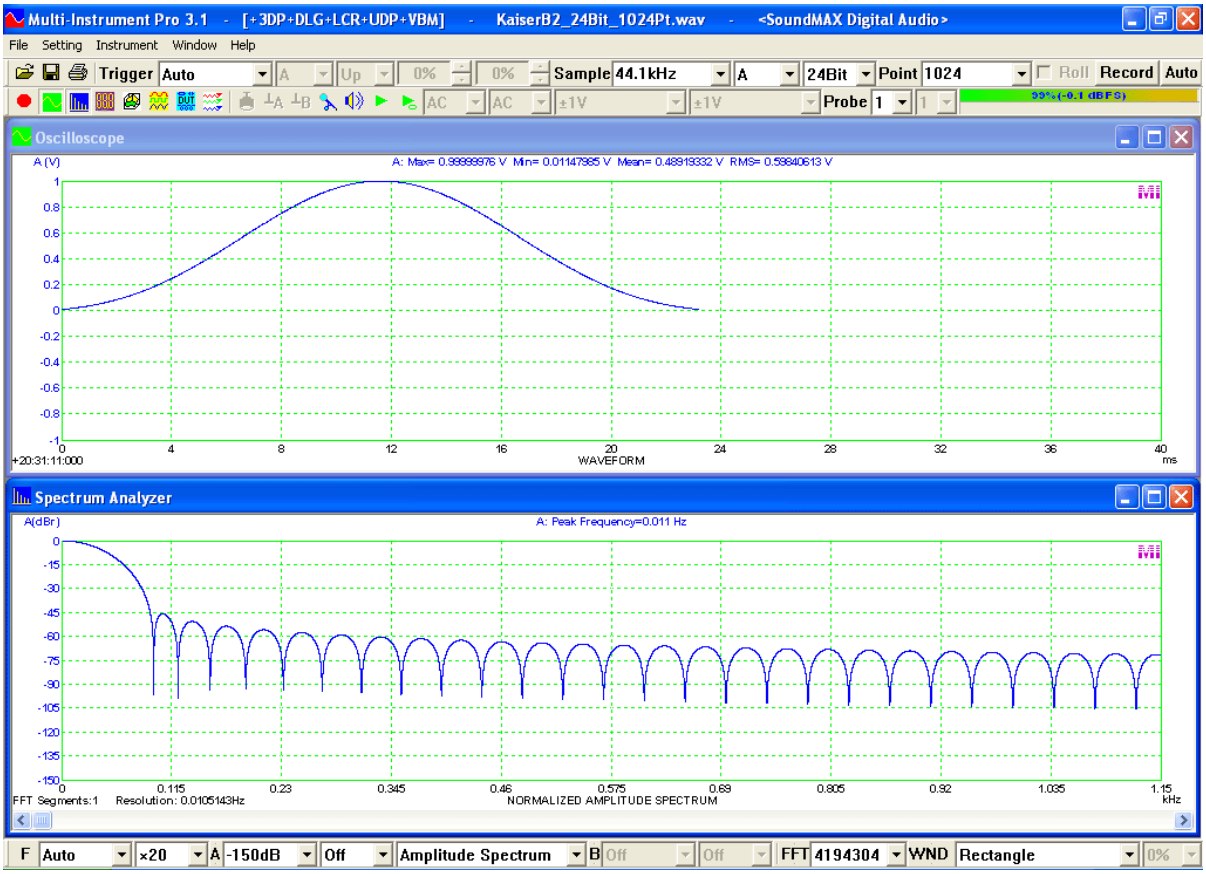
3.37 Kaiser-Bessel Window ($\alpha = 2.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 2.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-46	-6	1.43	1.99	1.45	0.49	1.50



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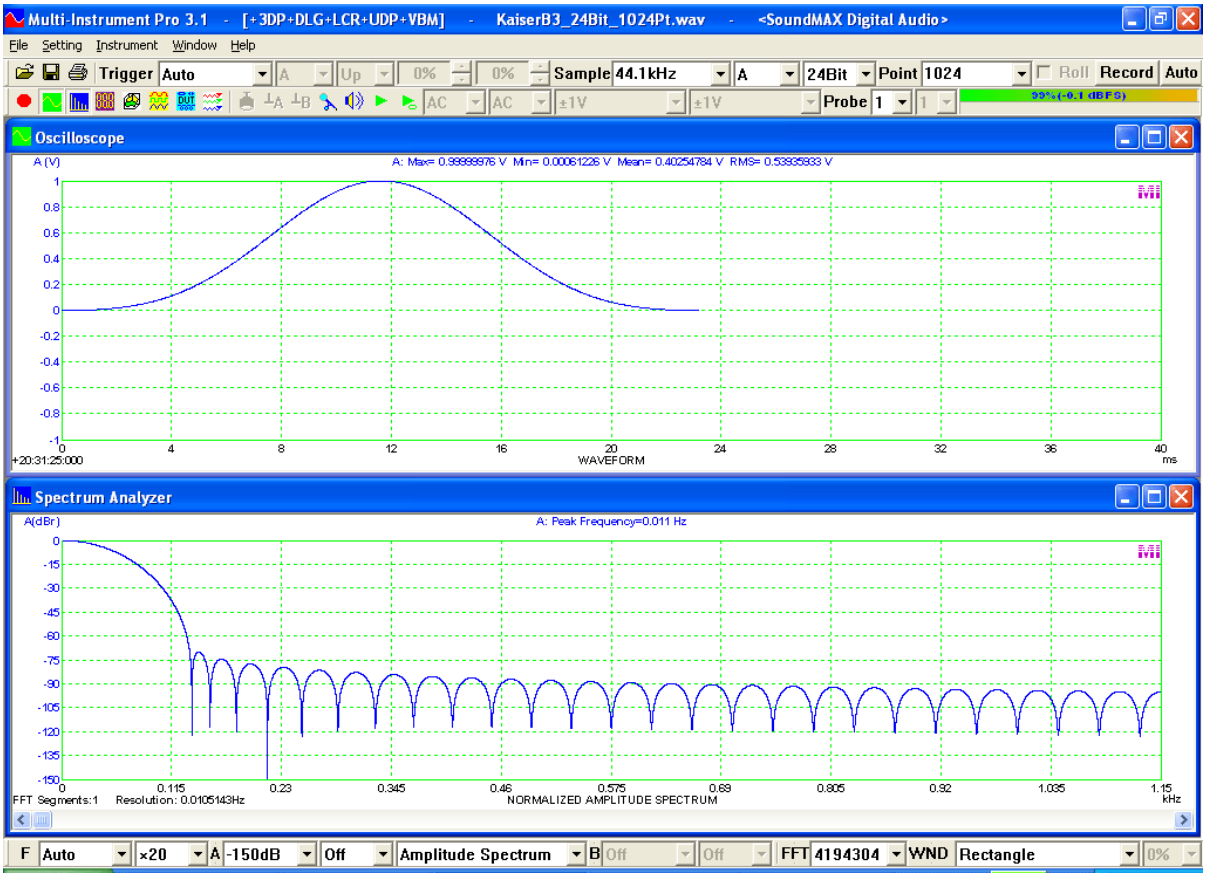
3.38 Kaiser-Bessel Window ($\alpha = 3.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 3.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-70	-6	1.70	2.39	1.02	0.40	1.80



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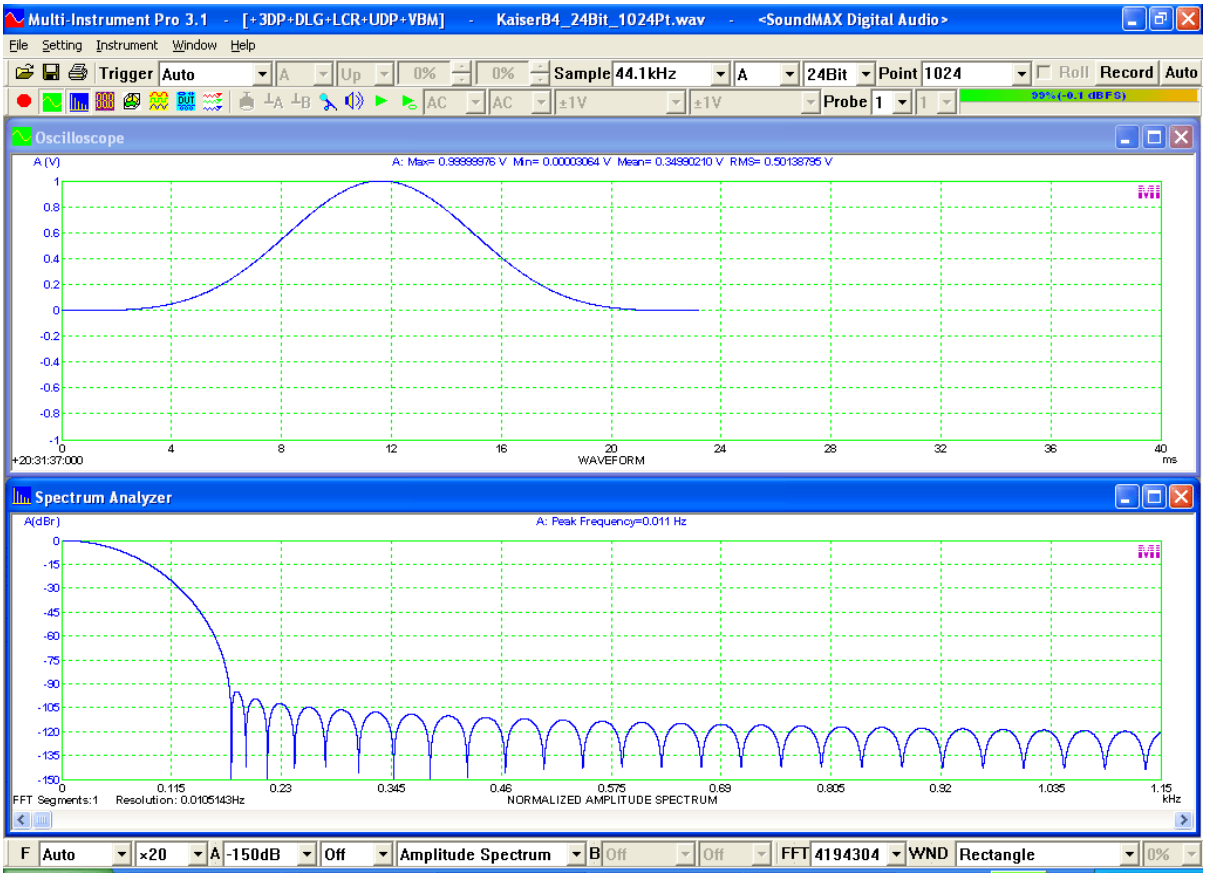
3.39 Kaiser-Bessel Window ($\alpha = 4.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 4.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-94	-6	1.94	2.73	0.79	0.35	2.05



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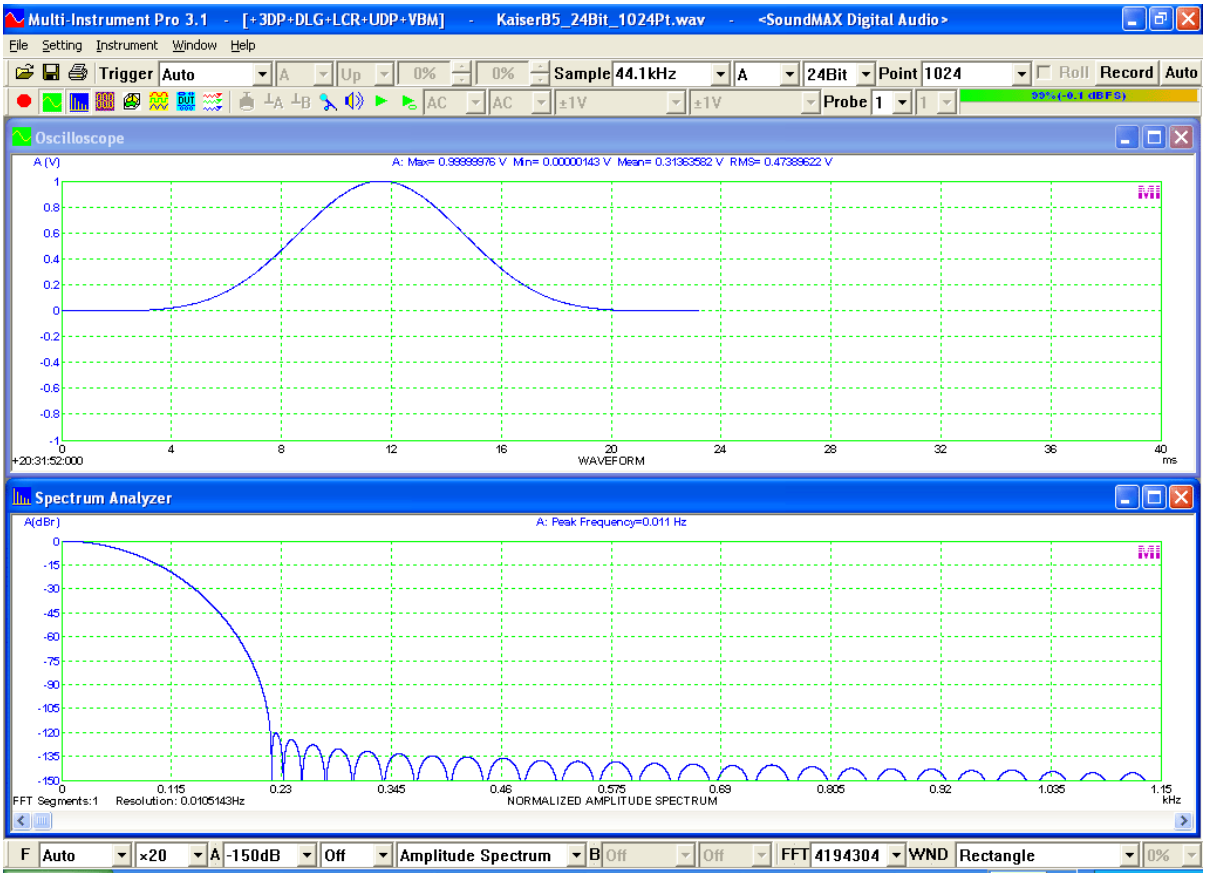
3.40 Kaiser-Bessel Window ($\alpha = 5.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where: bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 5.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-120	-6	2.16	3.03	0.64	0.31	2.28



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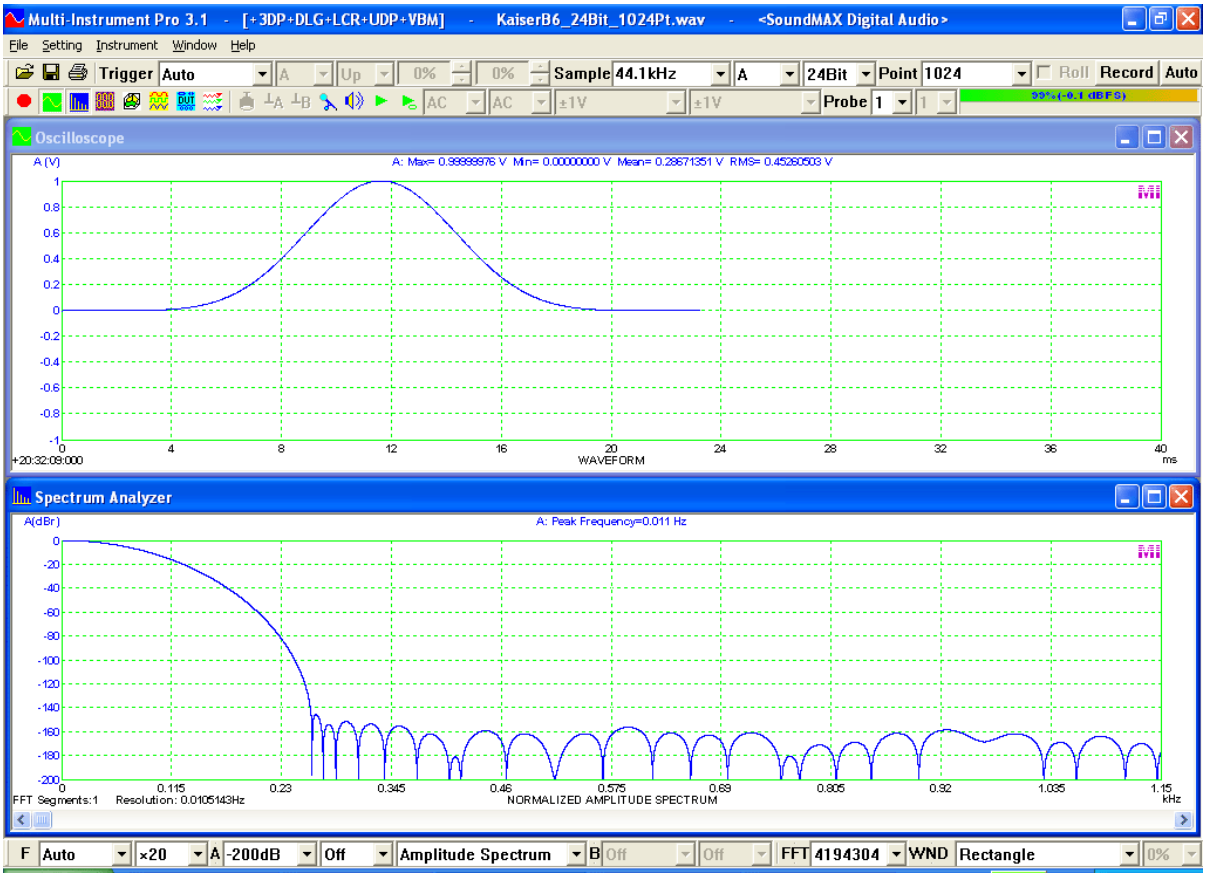
3.41 Kaiser-Bessel Window ($\alpha = 6.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 6.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-145	-6	2.35	3.31	0.54	0.29	2.49



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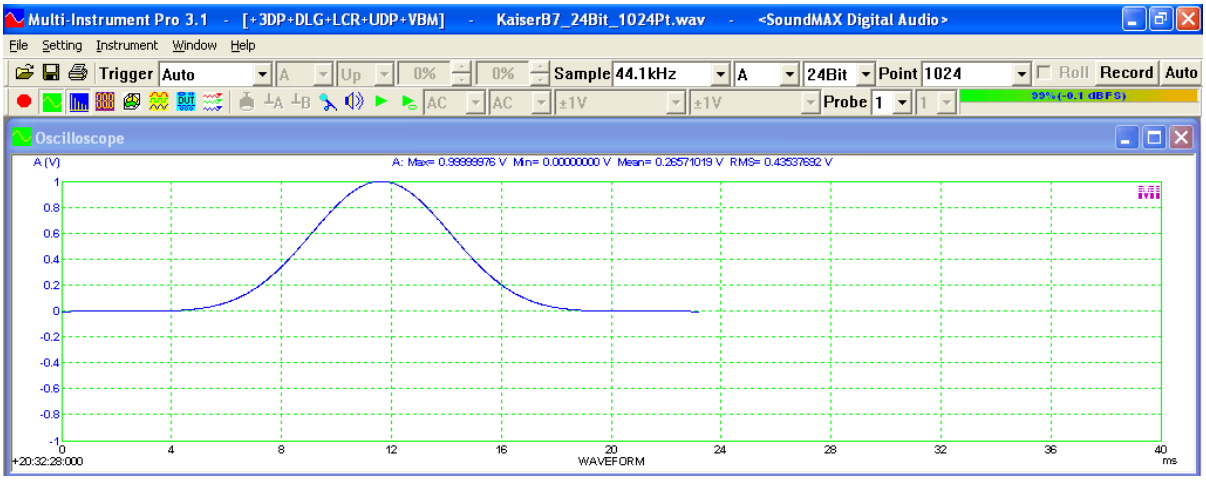
3.42 Kaiser-Bessel Window ($\alpha = 7.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 7.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-171	-6	2.53	3.56	0.47	0.27	2.68



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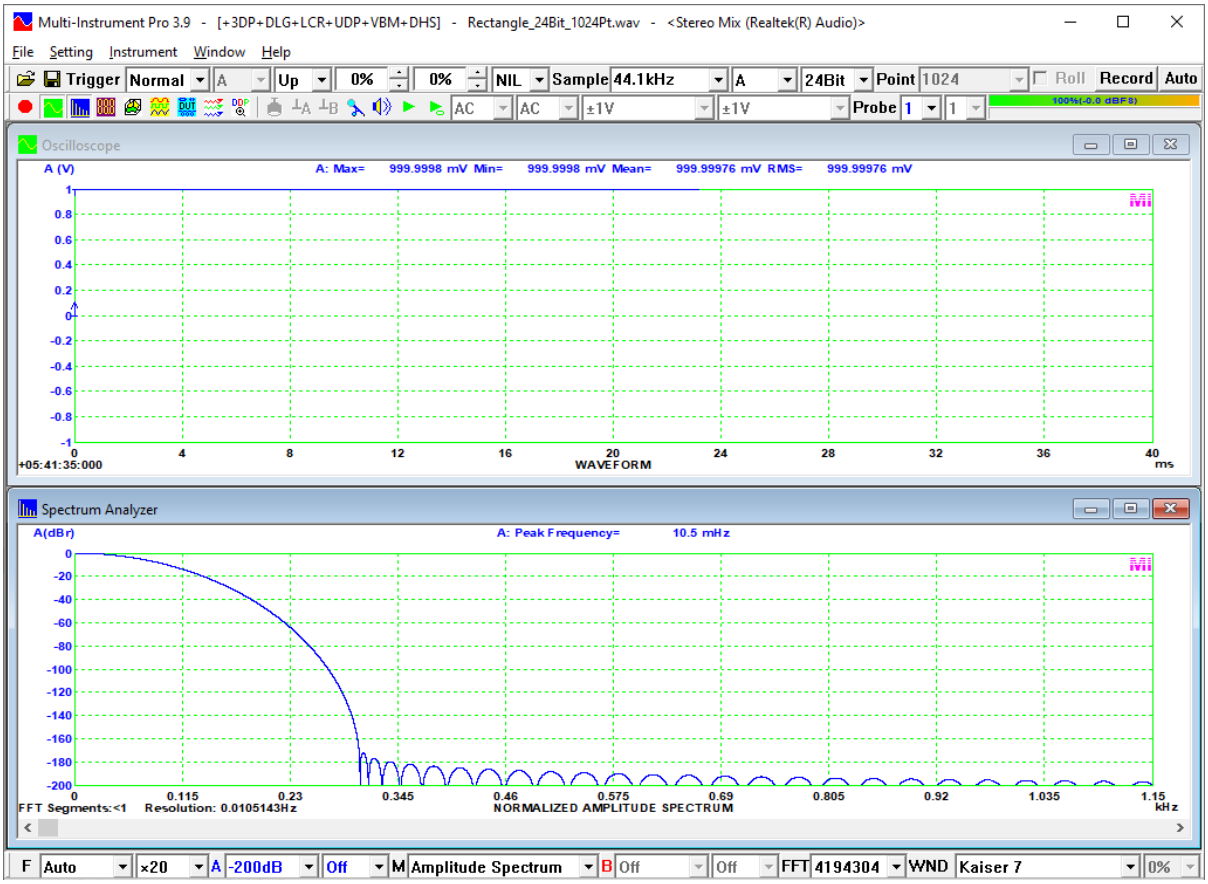

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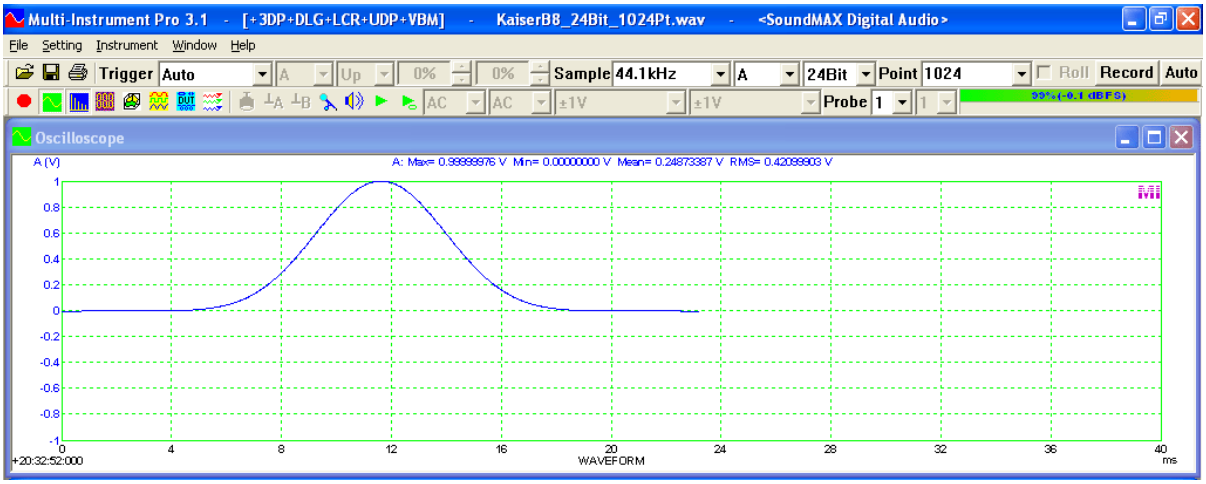
3.43 Kaiser-Bessel Window ($\alpha = 8.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 8.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-198	-6	2.70	3.80	0.41	0.25	2.86



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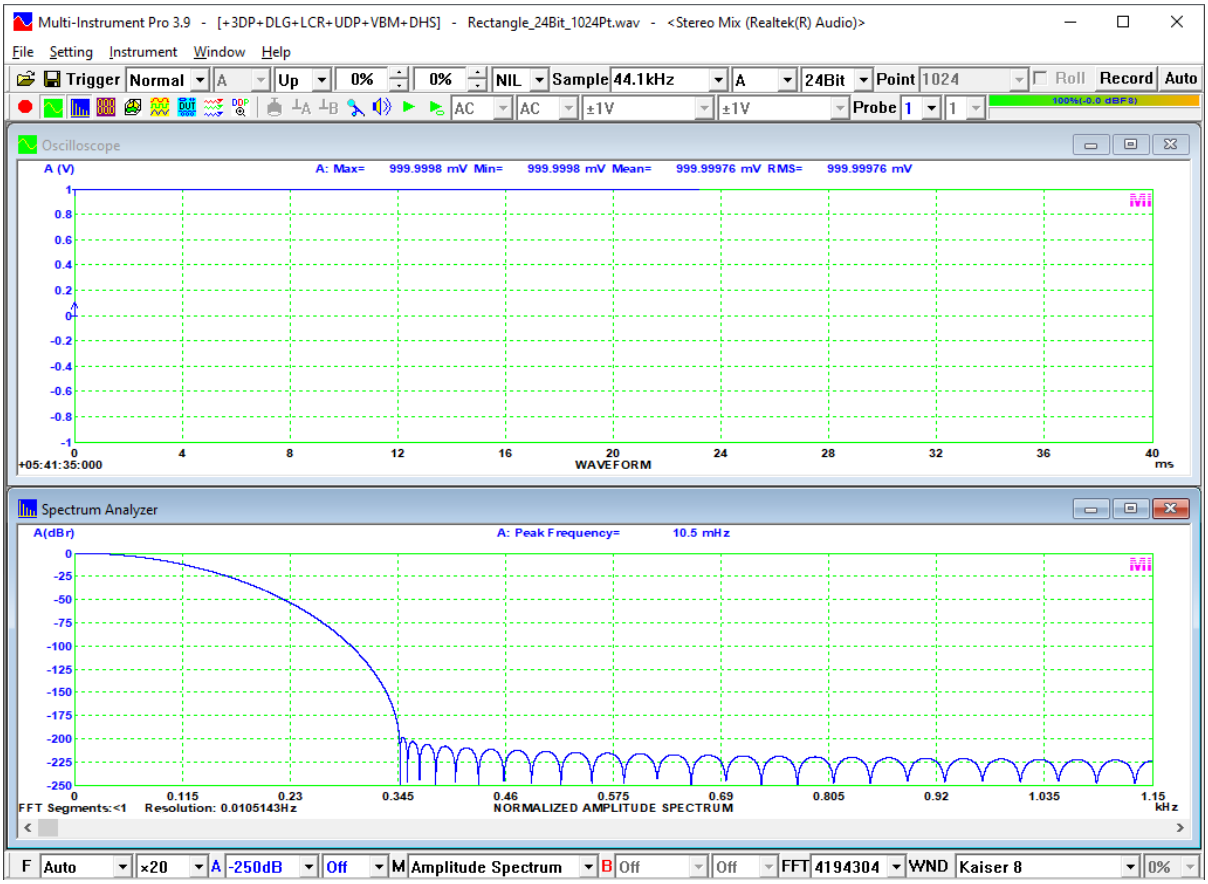

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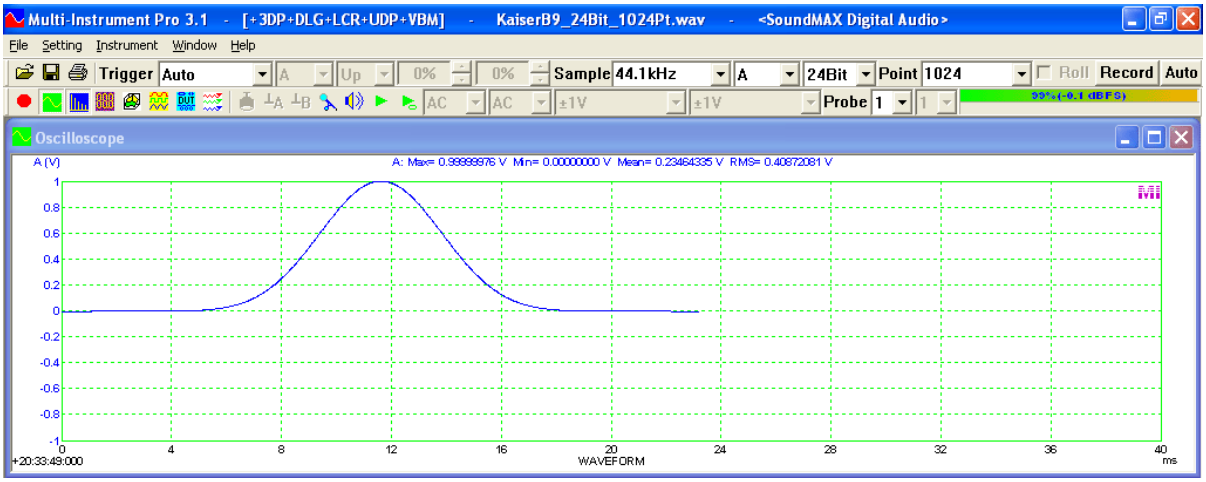
3.44 Kaiser-Bessel Window ($\alpha = 9.0$)

$w(n) = \text{bessi0}[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/\text{bessi0}[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$\text{bessel0}(x) = \Sigma\,[(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 9.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-224	-6	2.86	4.03	0.37	0.23	3.03



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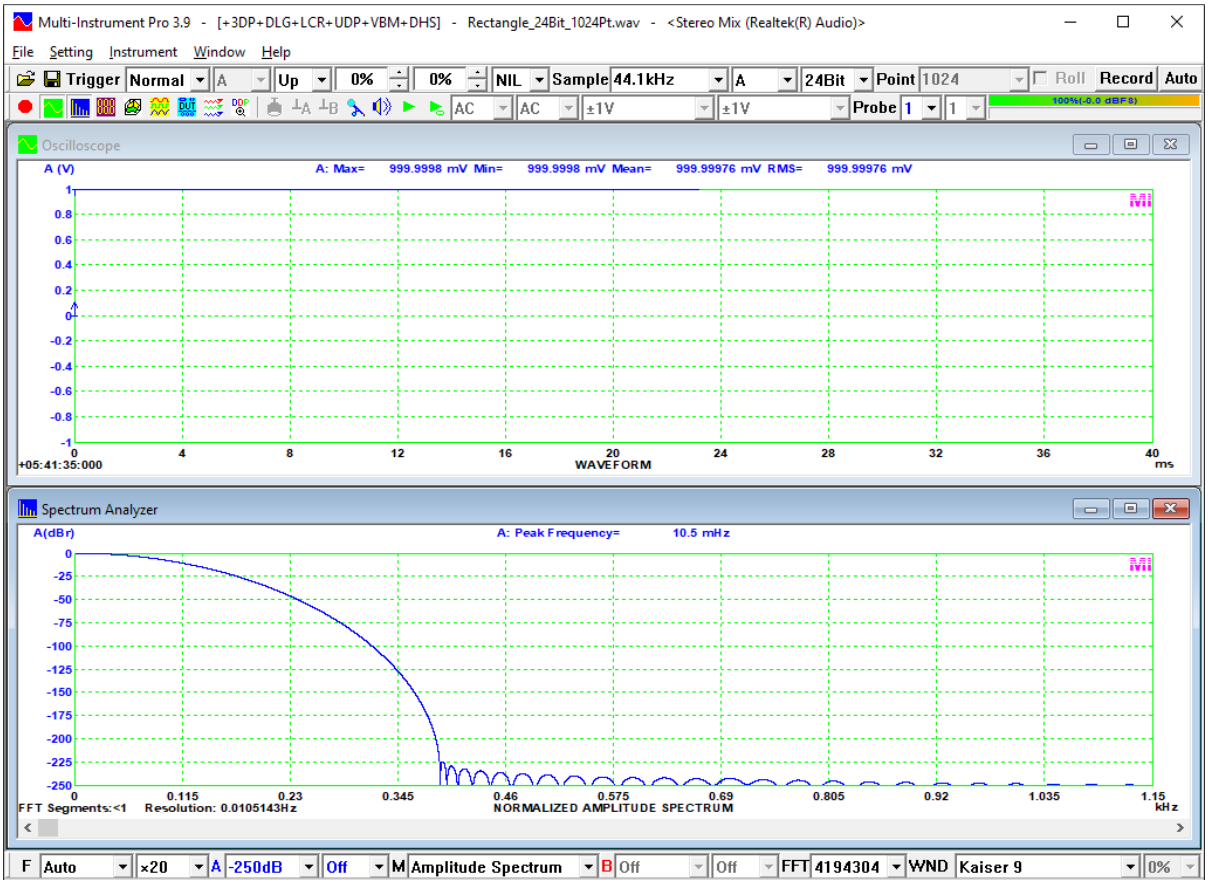

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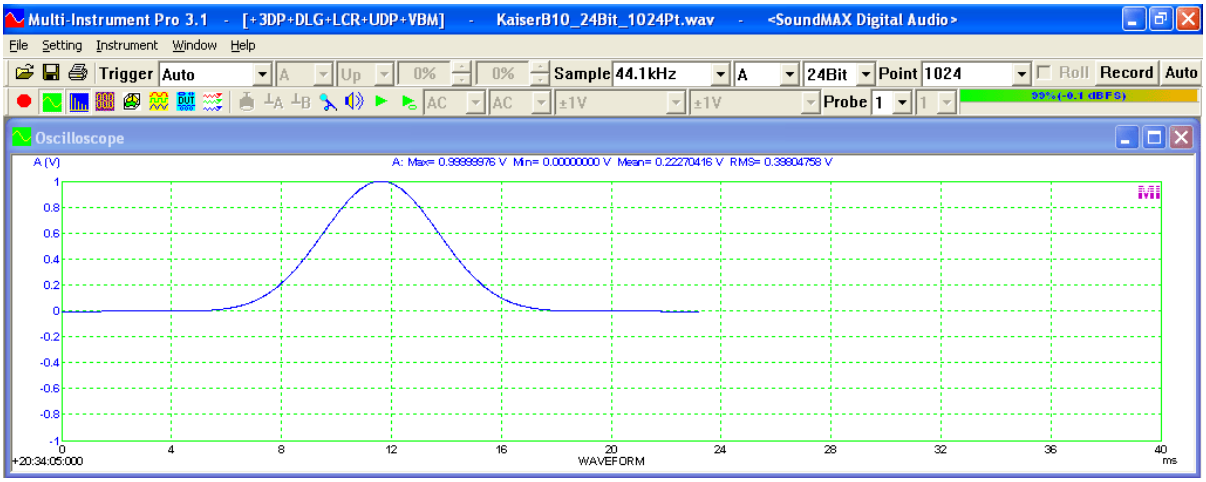
3.45 Kaiser-Bessel Window ($\alpha = 10.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 10.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-250	-6	3.01	4.24	0.33	0.22	3.19



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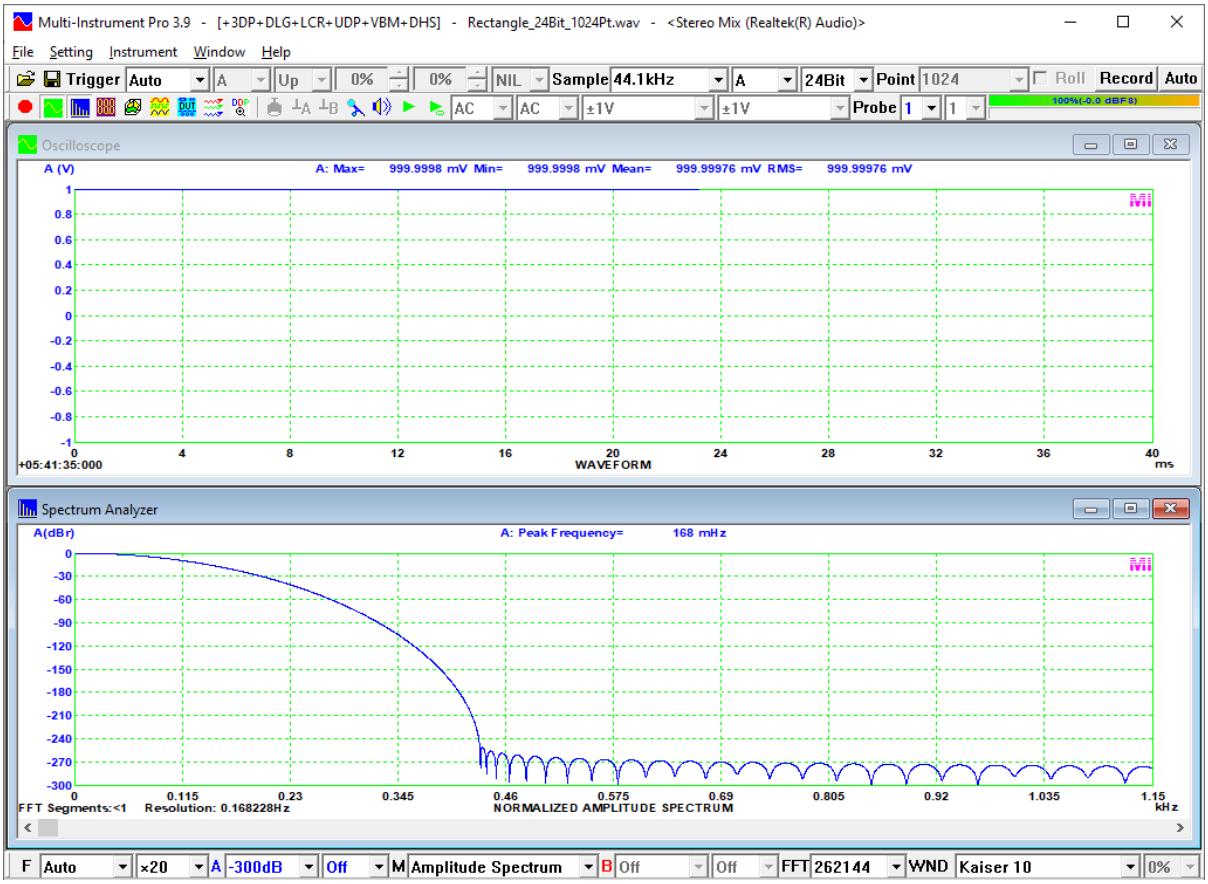

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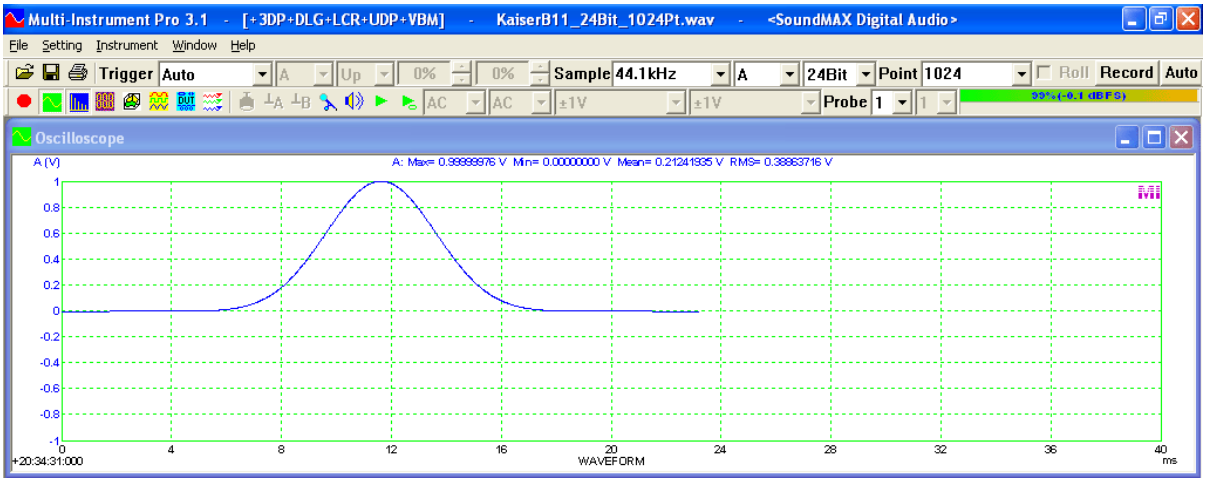
3.46 Kaiser-Bessel Window ($\alpha = 11.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 11.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-276	-6	3.15	4.44	0.30	0.21	3.35



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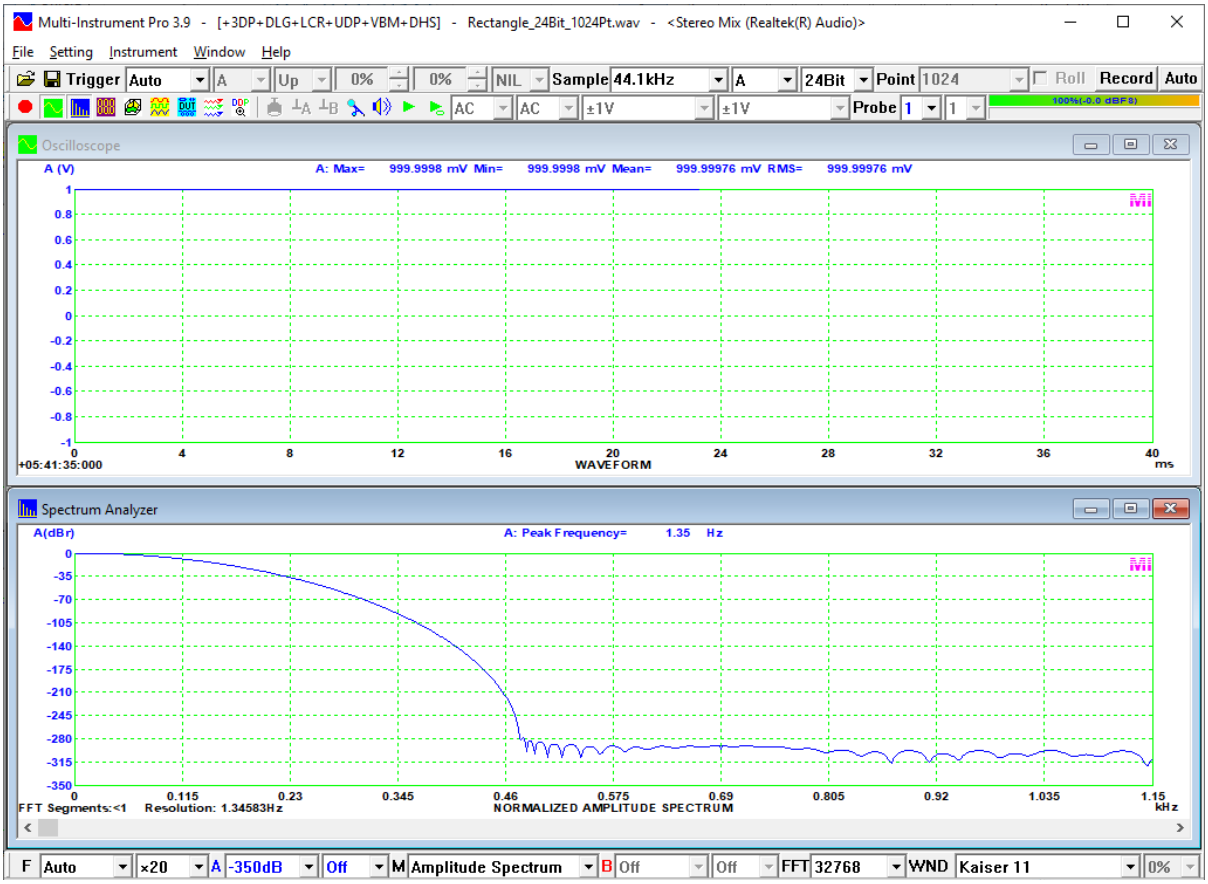

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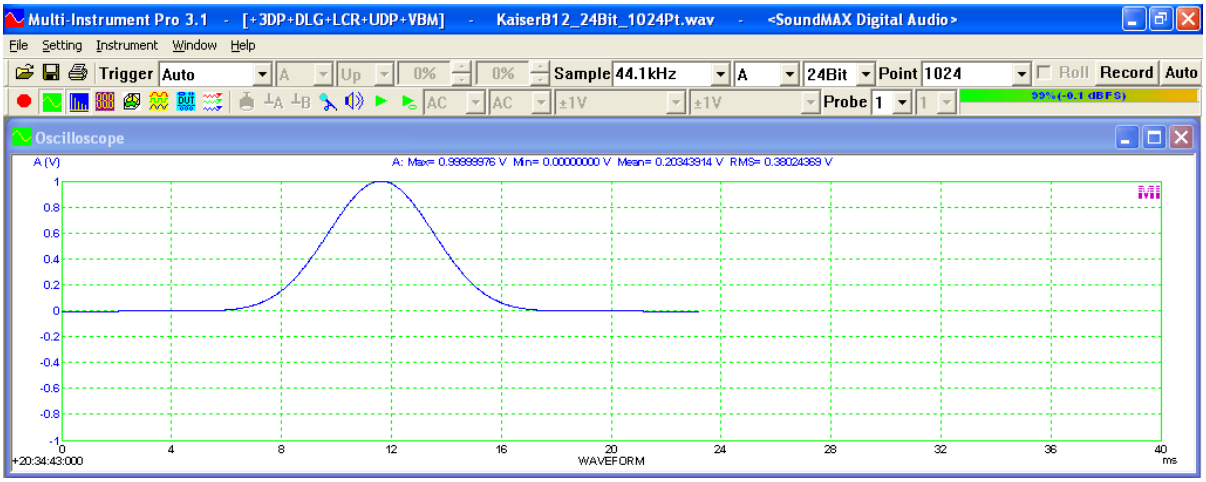
3.47 Kaiser-Bessel Window ($\alpha = 12.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 12.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-306	-6	3.29	4.64	0.28	0.20	3.49



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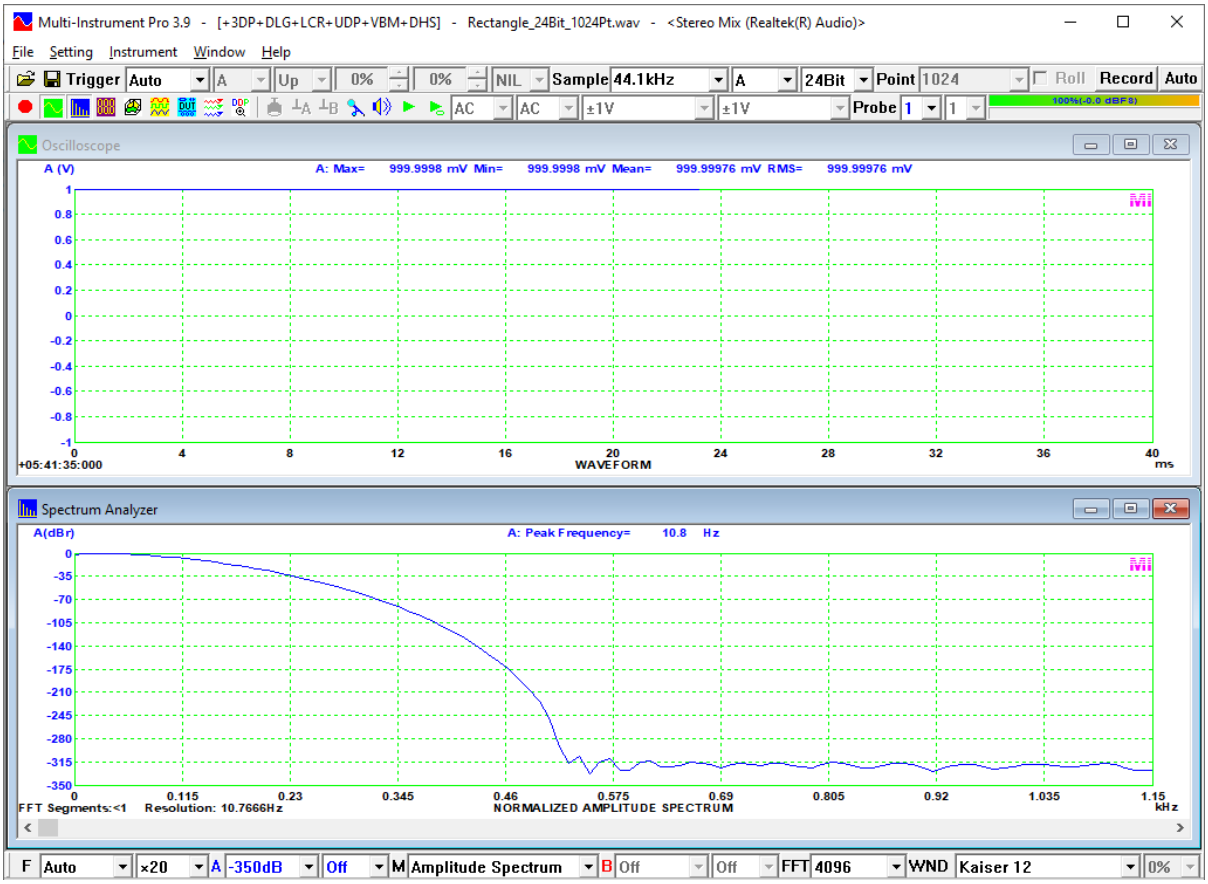

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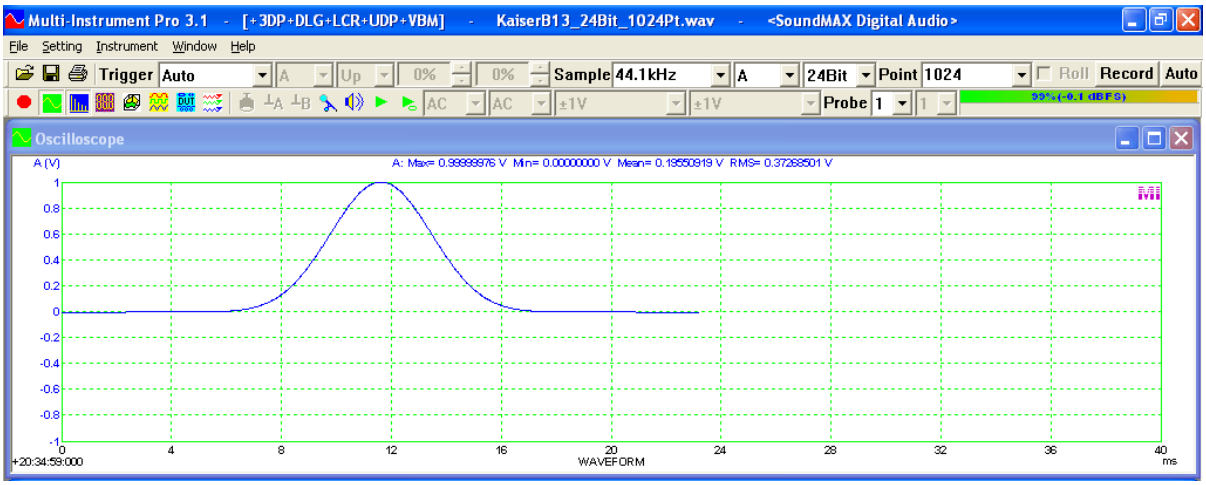
3.48 Kaiser-Bessel Window (α = 13.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 13.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.42	4.82	0.26	0.20	3.63



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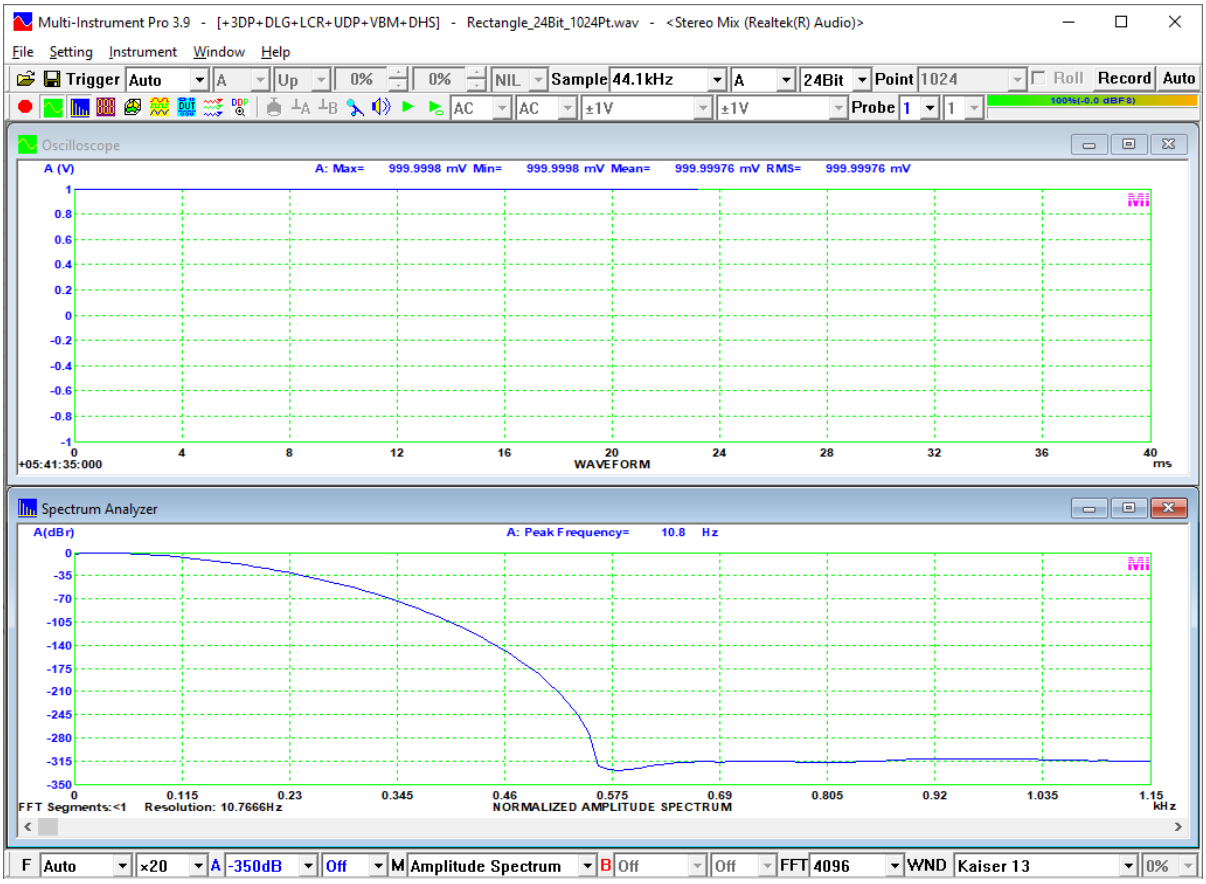

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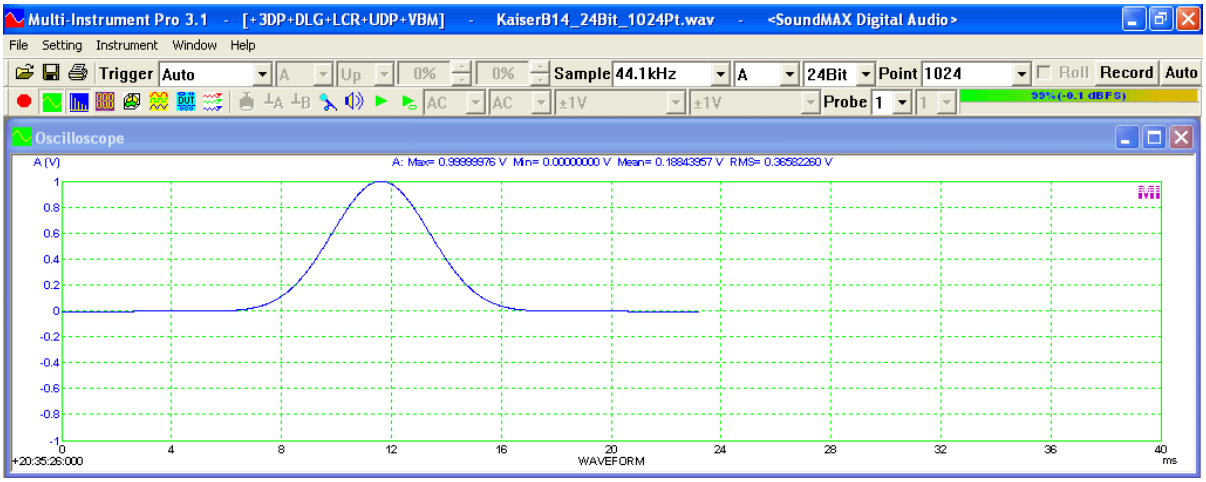
3.49 Kaiser-Bessel Window ($\alpha = 14.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 14.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.54	5.0	0.24	0.19	3.77



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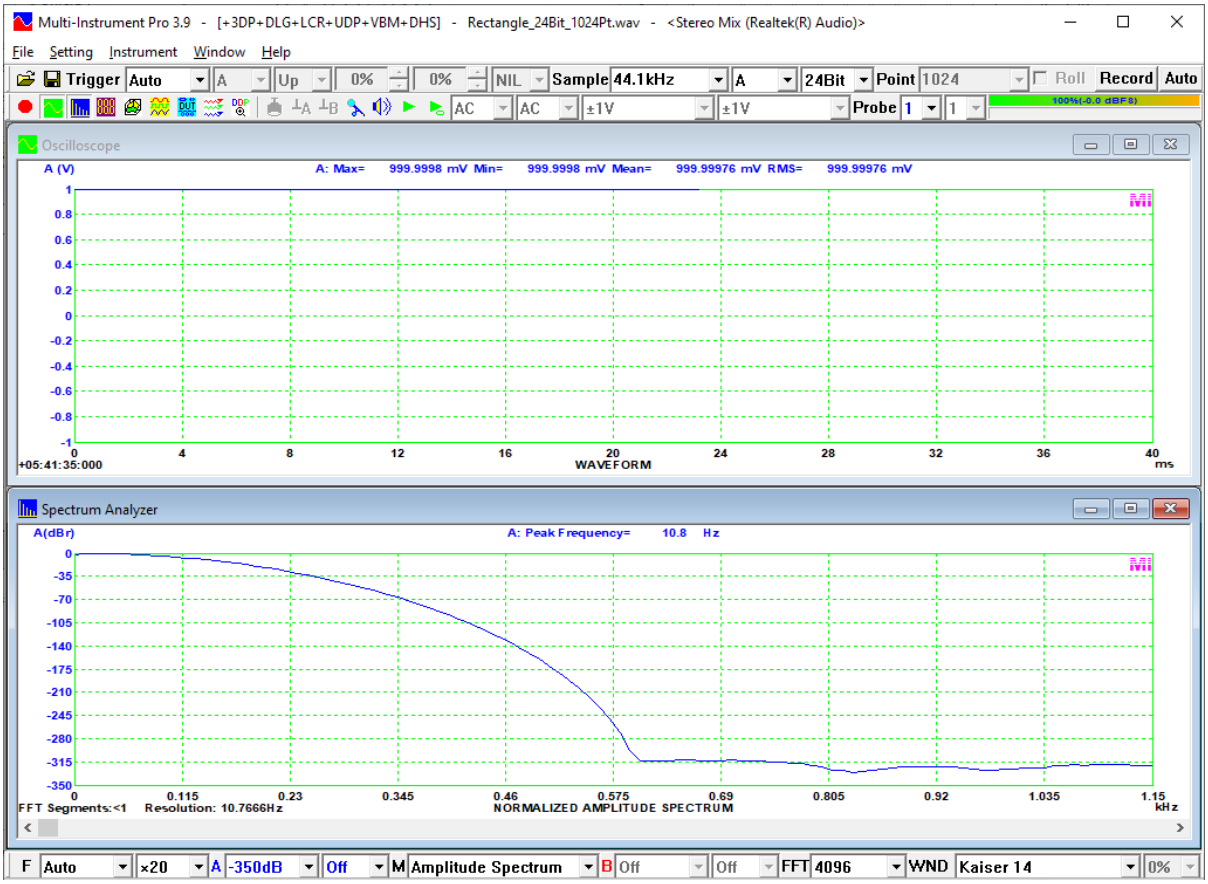

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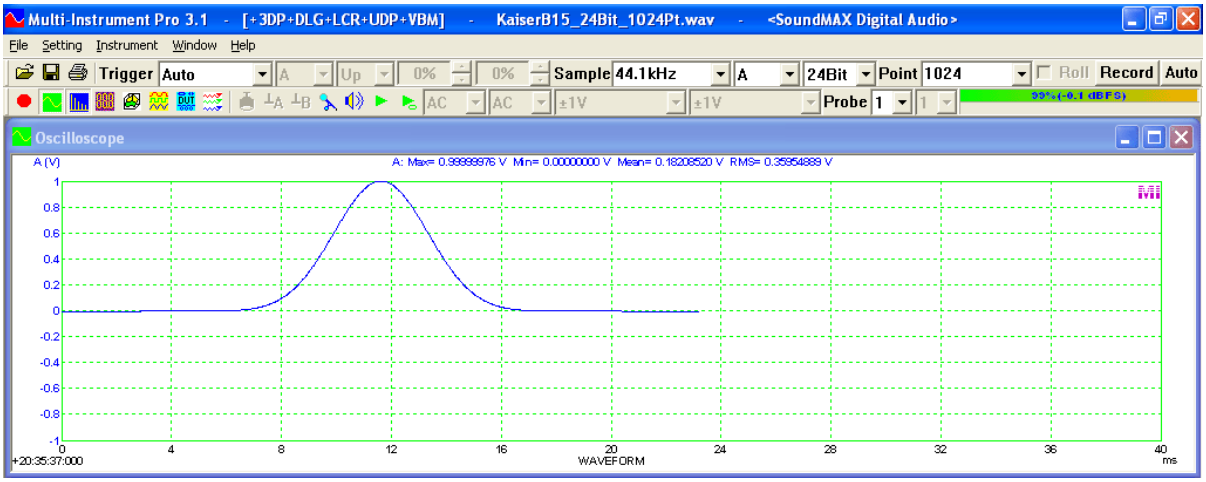
3.50 Kaiser-Bessel Window ($\alpha = 15.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 15.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.66	5.17	0.22	0.18	3.90



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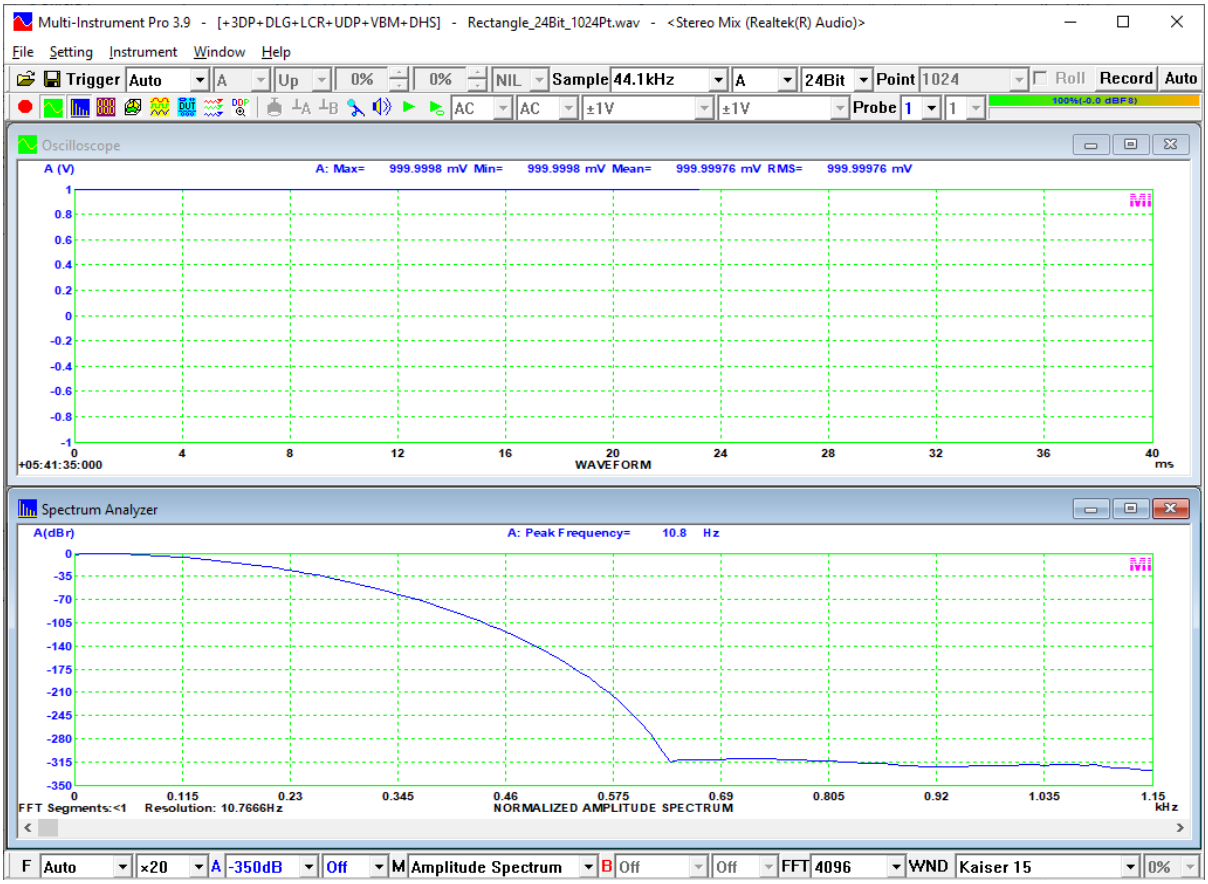

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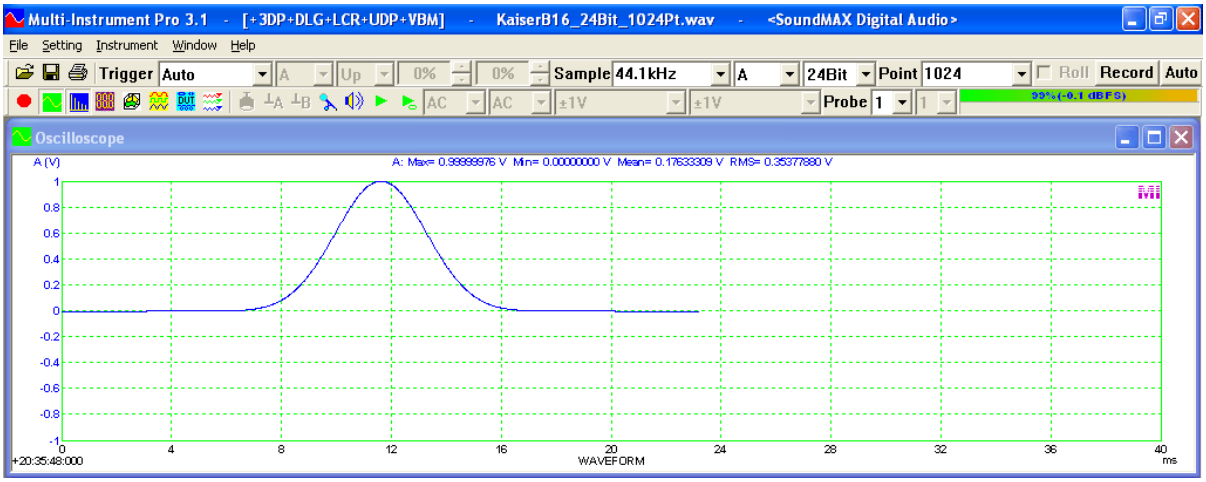
3.51 Kaiser-Bessel Window ($\alpha = 16.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 16.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.79	5.34	0.21	0.18	4.03



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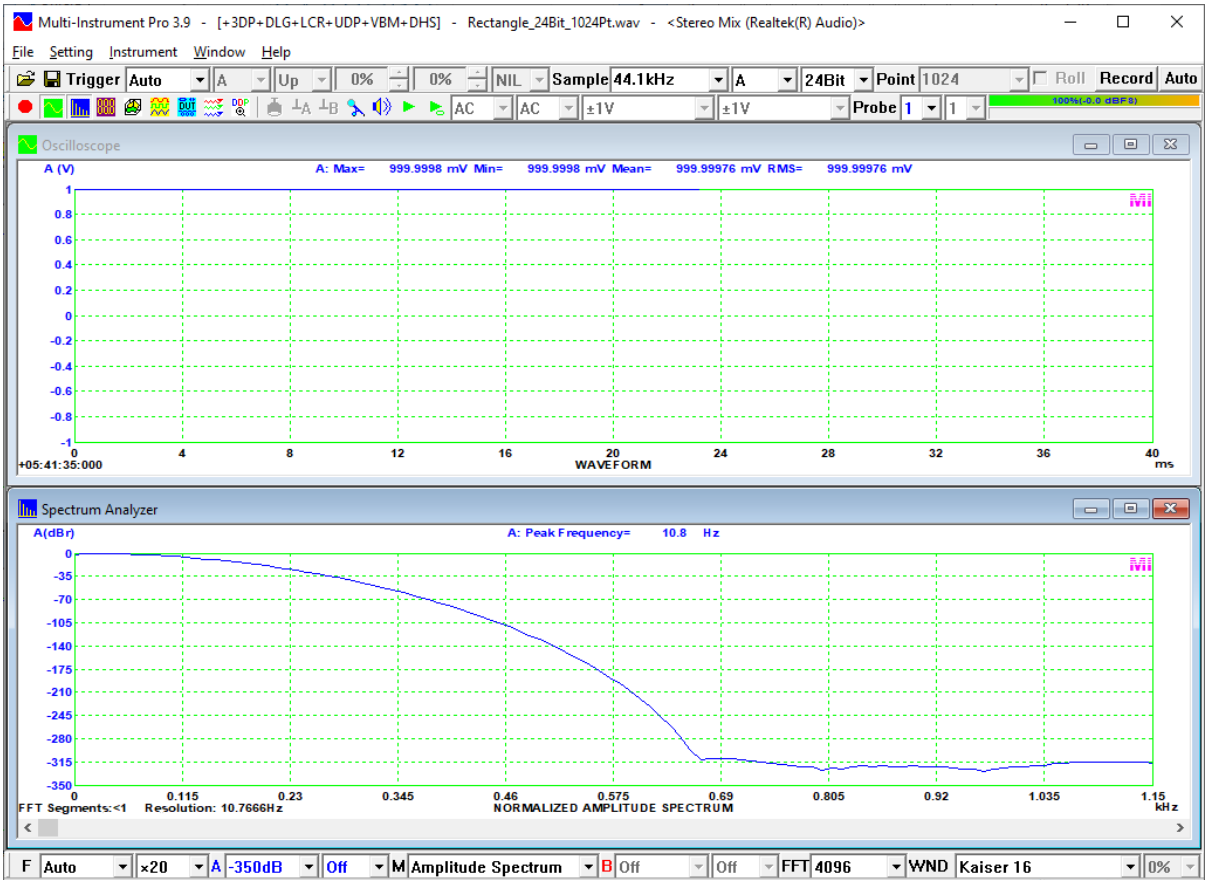

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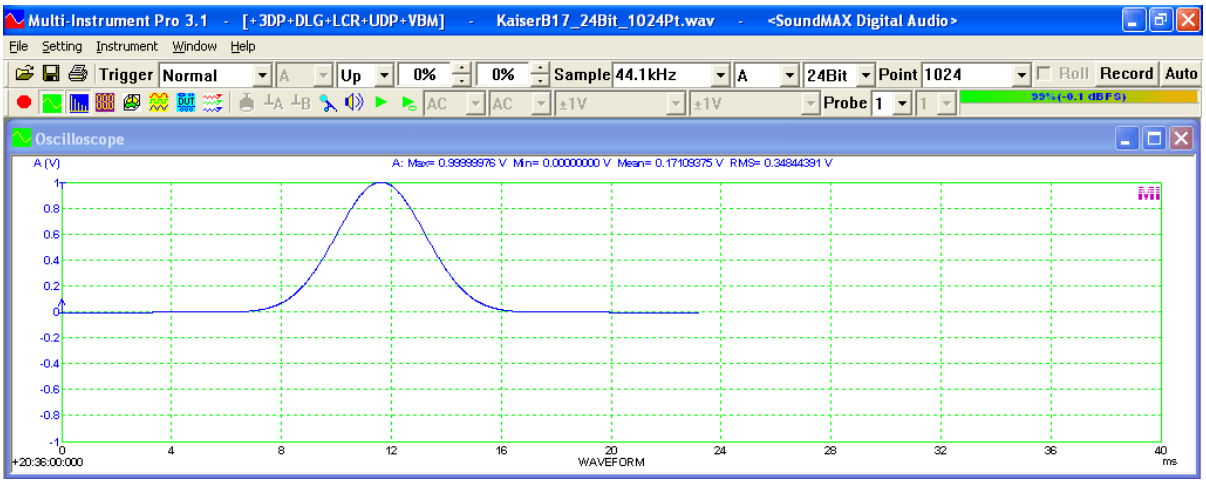
3.52 Kaiser-Bessel Window (α = 17.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 17.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	3.90	5.50	0.20	0.17	4.15



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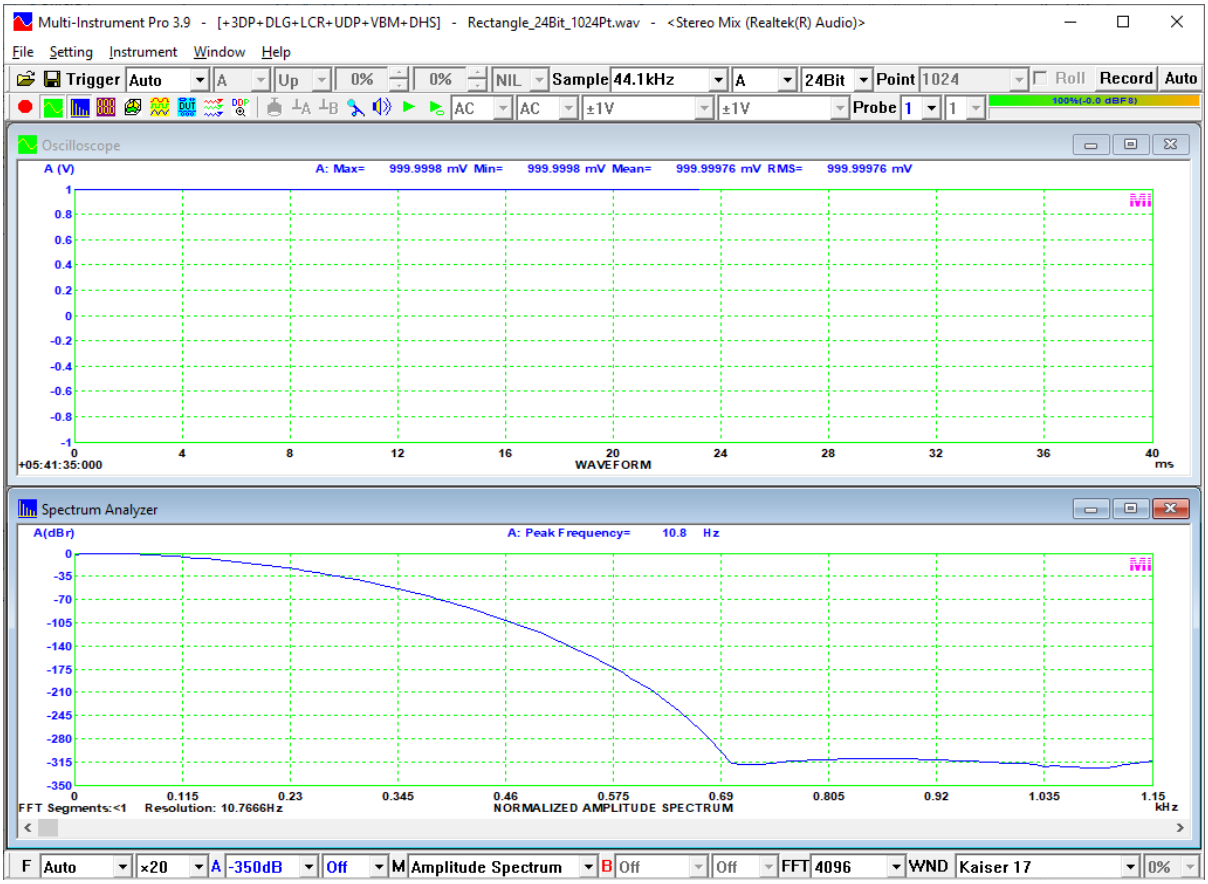

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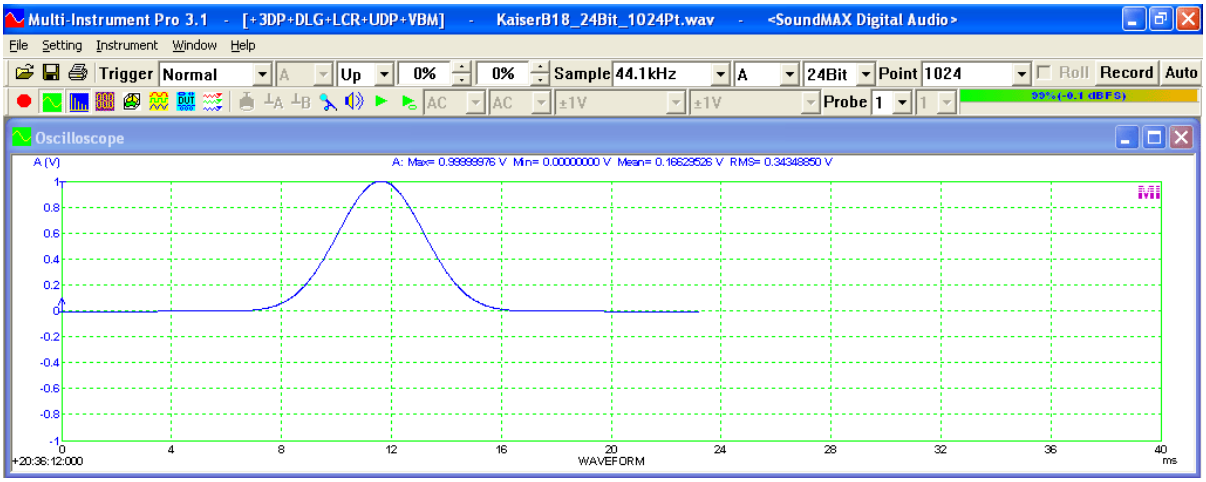
3.53 Kaiser-Bessel Window ($\alpha = 18.0$)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where $k = 0 \sim \infty$.
$n = 0,1,\ldots,N-1$;

$\alpha = 18.0$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	4.01	5.66	0.19	0.17	4.27



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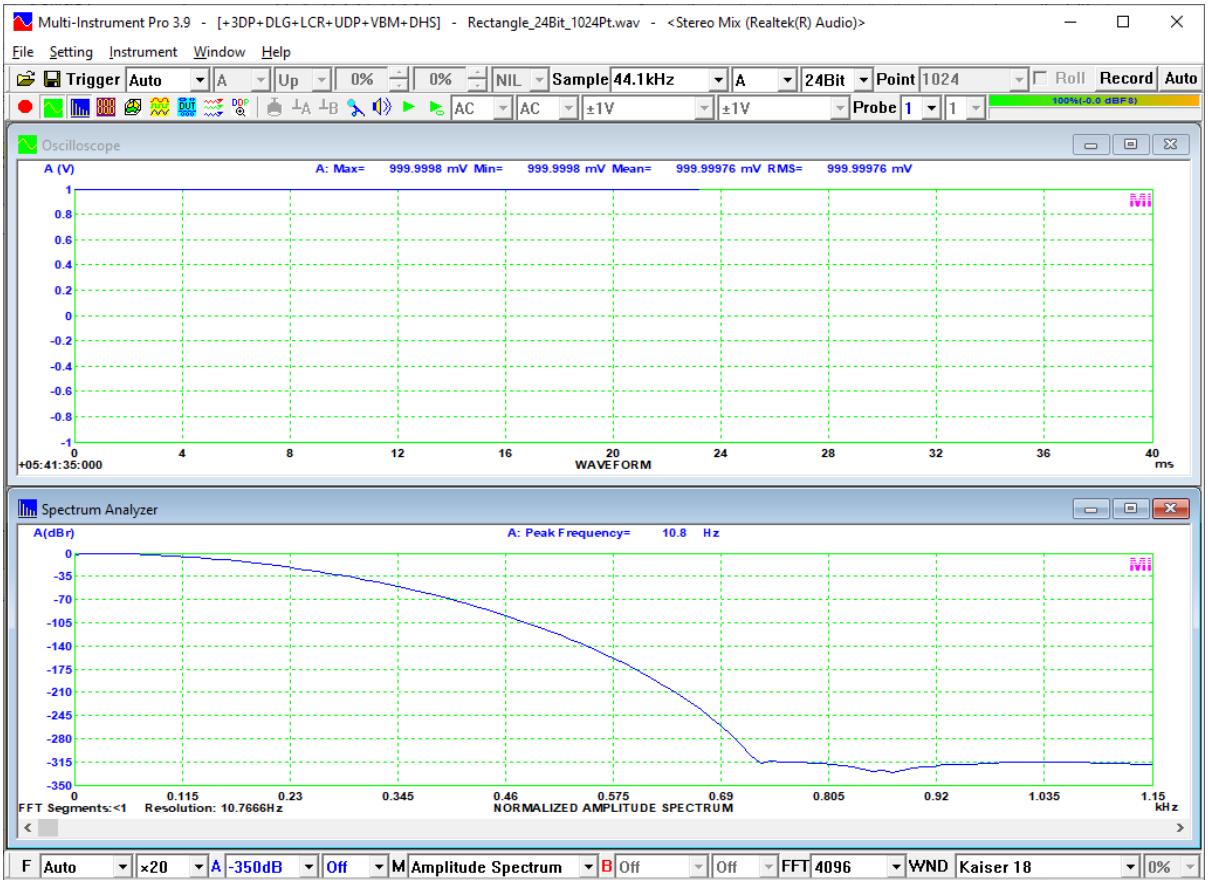

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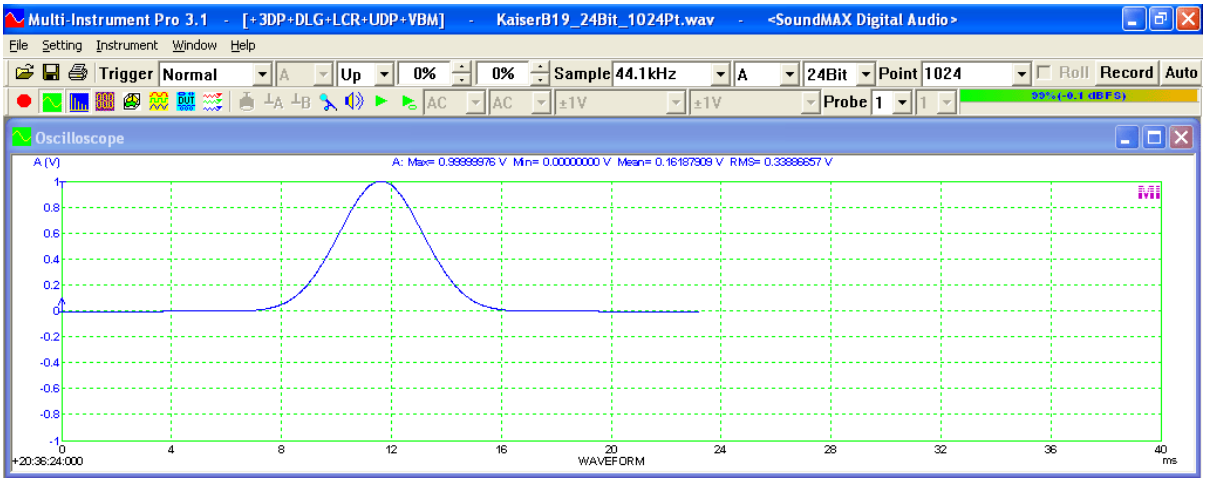
3.54 Kaiser-Bessel Window (α = 19.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 19.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	4.12	5.81	0.18	0.16	4.38



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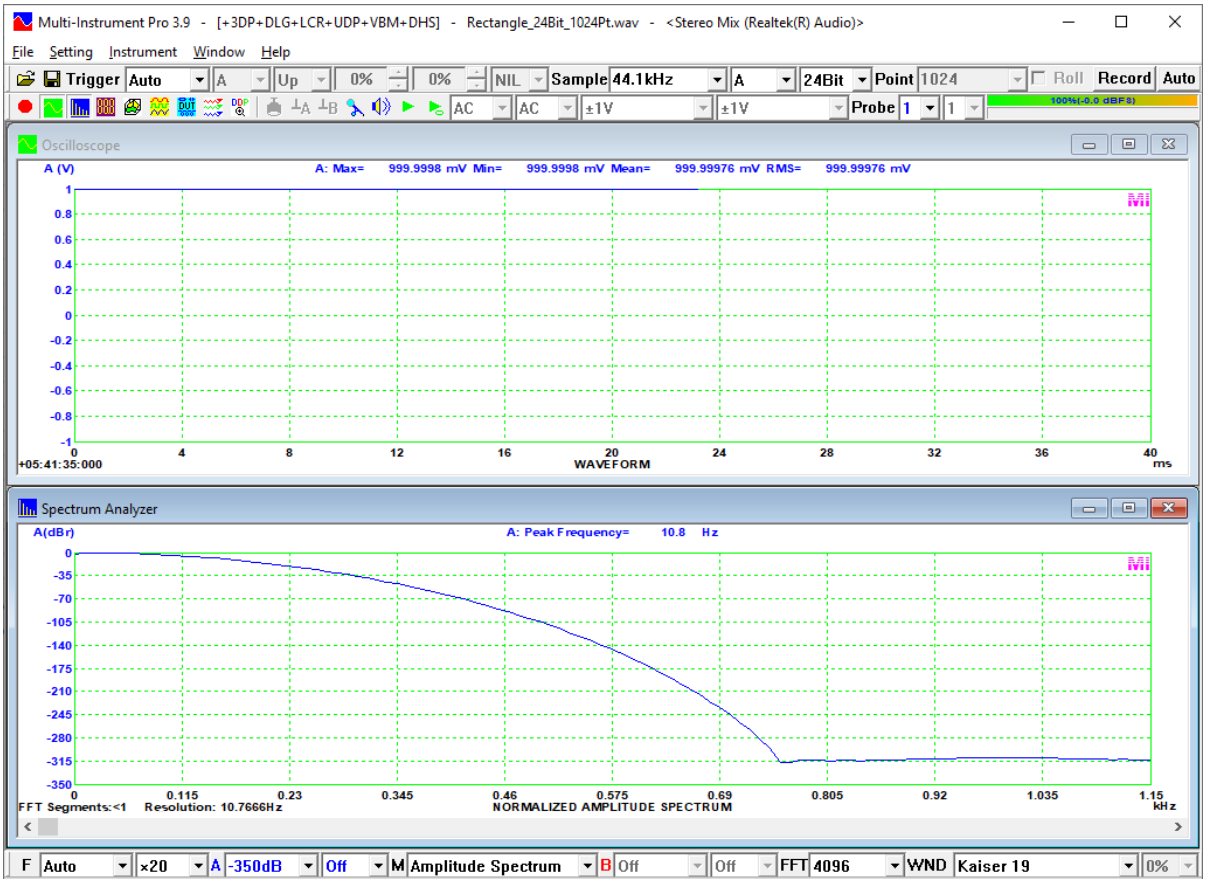

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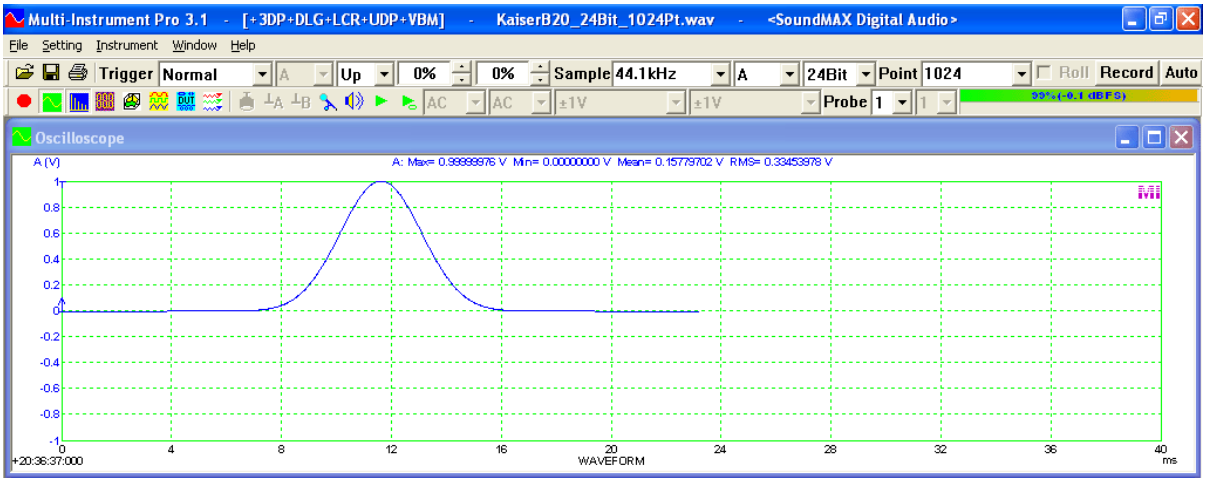
3.55 Kaiser-Bessel Window (α = 20.0)

$w(n) = bessi0[\alpha\pi(1 - (2n/N - 1)^2)^{0.5}]/bessi0[\alpha\pi]$

where:
bessi0 is the zero-order modified Bessel function of the first kind.
$bessel0(x) = \Sigma [(x/2)^k / k!]^2$, where k= 0 ~ ∞.
n = 0,1,…,N-1;

α = 20.0

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
<-325	-6	4.23	5.96	0.17	0.16	4.49



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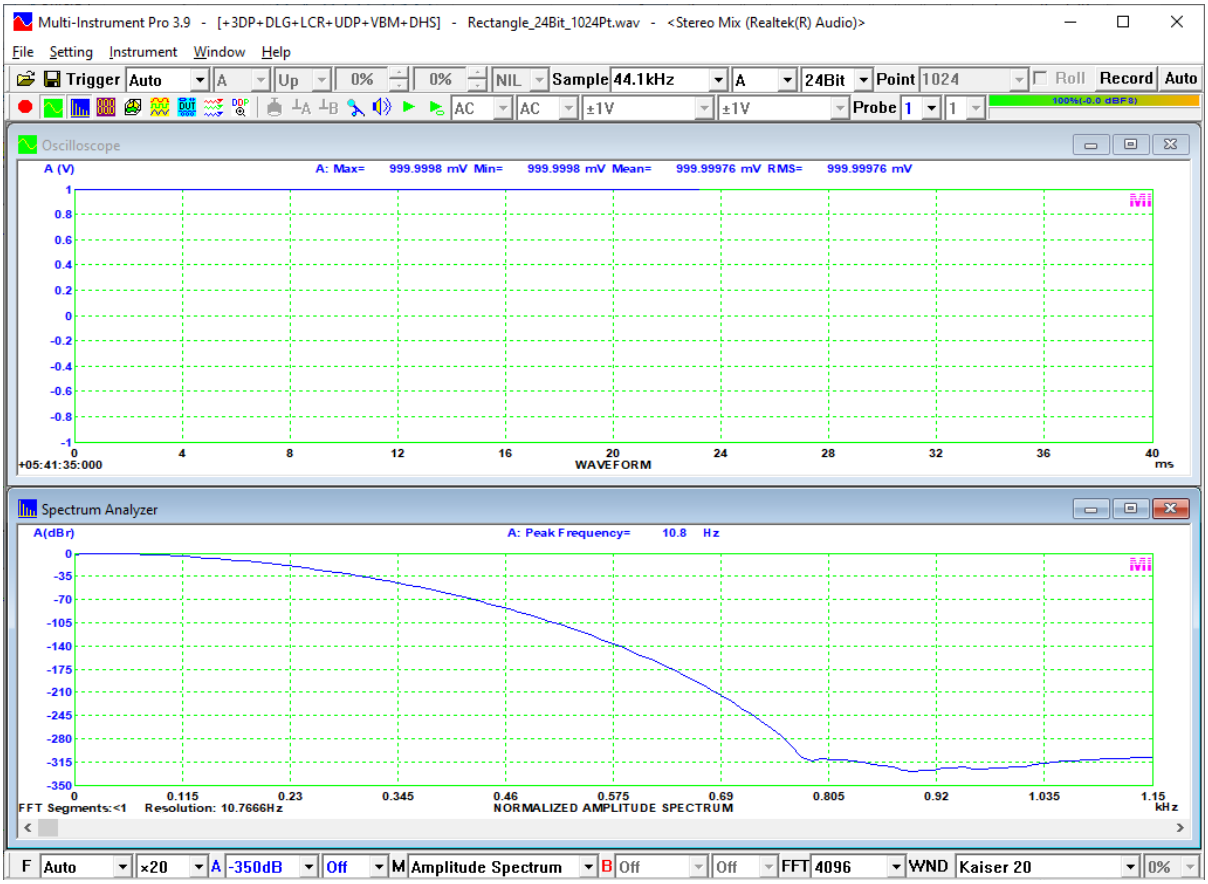

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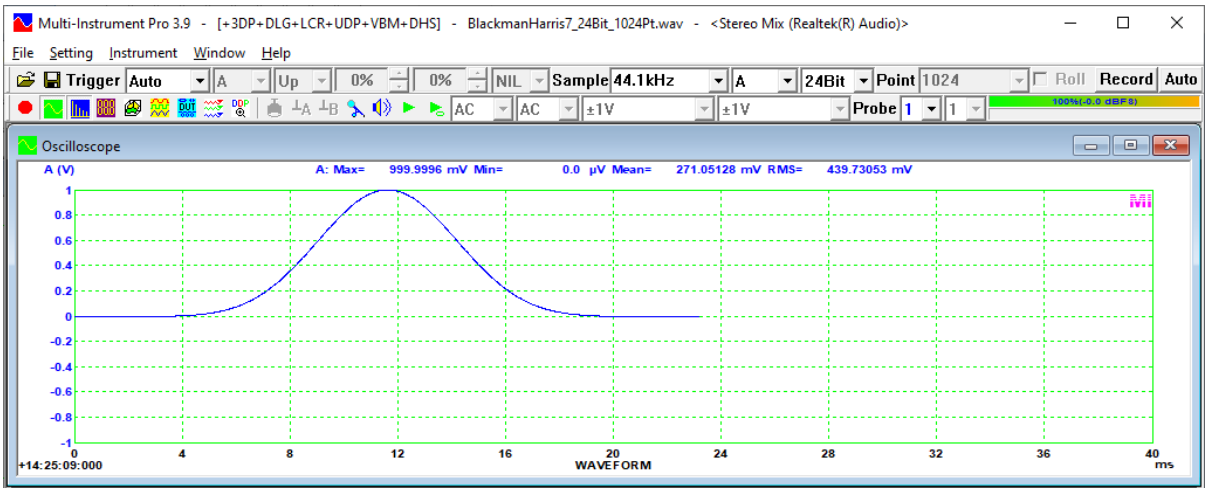
3.56 Blackman-Harris (7 terms) Window

$$w(n) = 0.27105140069342 - 0.43329793923448 \times \cos(2n\pi/N) + 0.21812299954311 \times \cos(4n\pi/N) - 0.06592544638803 \times \cos(6n\pi/N) + 0.01081174209837 \times \cos(8n\pi/N) - 0.00077658482522 \times \cos(10n\pi/N) + 0.00001388721735 \times \cos(12n\pi/N)$$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-180	<-6	2.48	3.50	0.48	0.27	2.63



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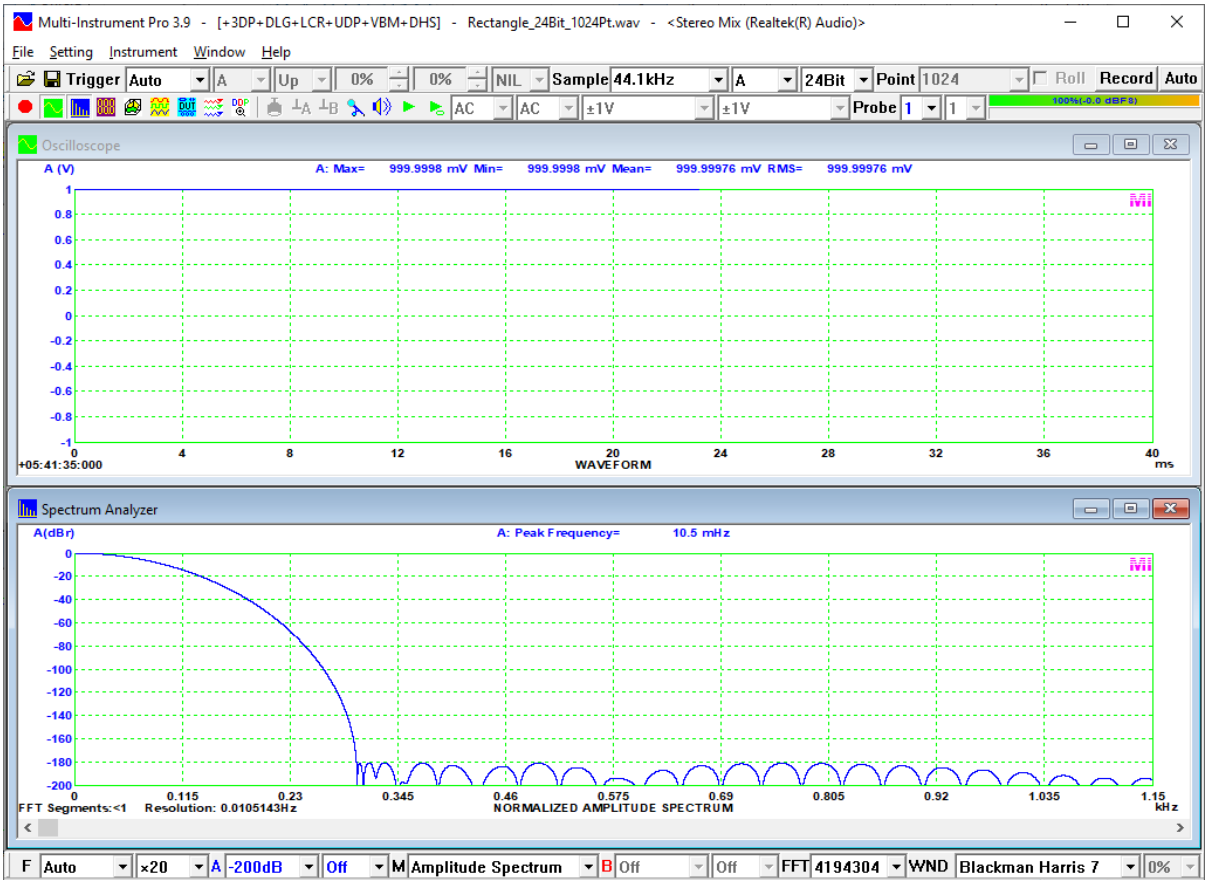

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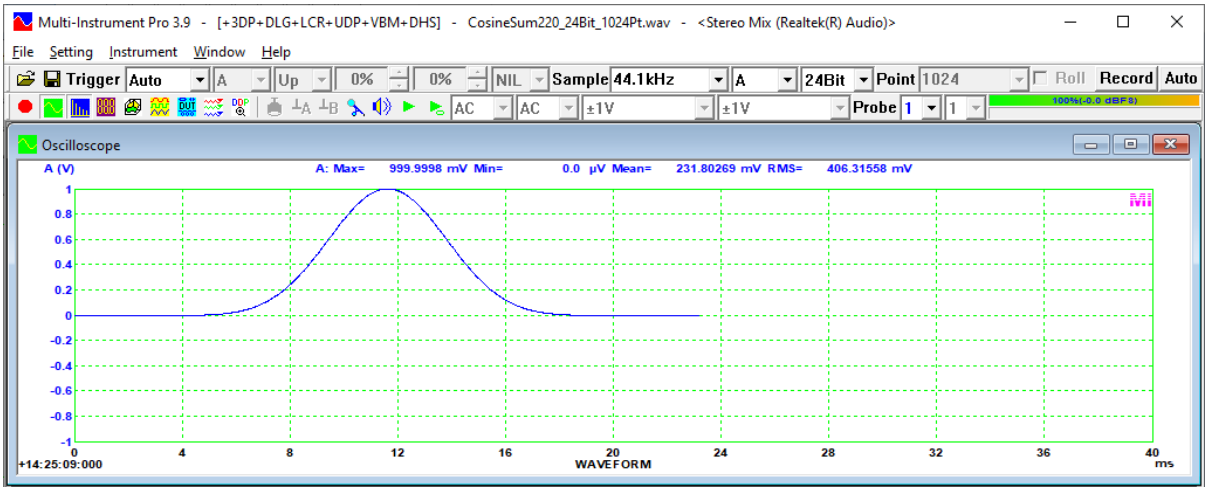
3.57 Cosine Sum 220 Window

$$w(n) = 0.231802801359030602839 3 - 0.393257547178948861508 1 \times \cos(2n\pi/N) + 0.238543476497074742945 4 \times \cos(4n\pi/N) - 0.101437043778523981126 8 \times \cos(6n\pi/N) + 0.029115160619180039186 45 \times \cos(8n\pi/N) - 0.005280988177252078698806 \times \cos(10n\pi/N) + 0.000538290909338194536352 8 \times \cos(12n\pi/N) - 0.000024420865275078677301 68 \times \cos(14n\pi/N) + 0.000000270615376420504353 2817 \times \cos(16n\pi/N)$$

$$n = 0,1,\ldots,N-1;$$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-220	-36	2.90	4.08	0.36	0.23	3.07



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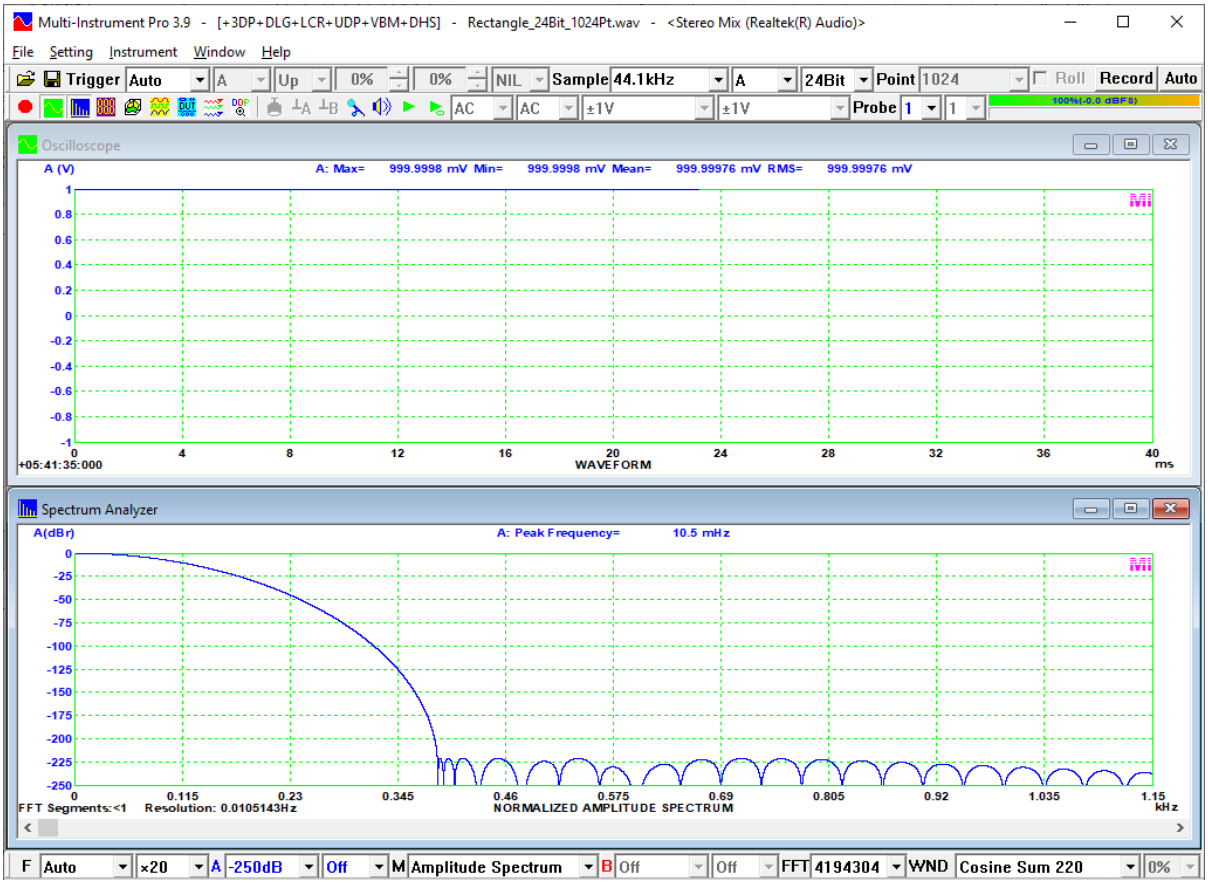

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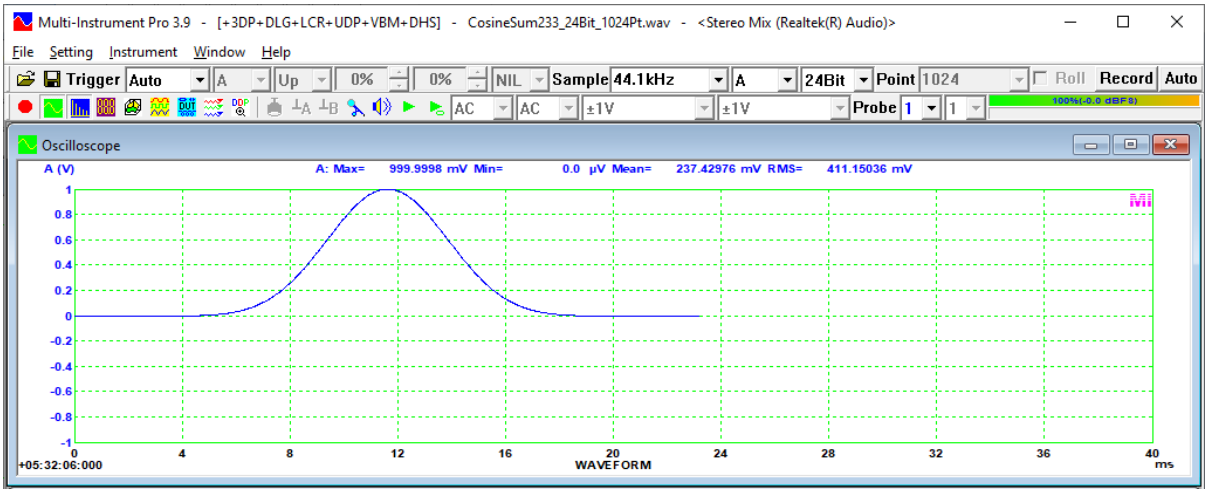
3.58 Cosine Sum 233 Window

$$w(n) = 0.237429874153246592822 6 - 0.399470437380100935800 1 \times \cos(2n\pi/N) + 0.236264460810028247513 3 \times \cos(4n\pi/N) - 0.096206768383635166490 24 \times \cos(6n\pi/N) + 0.025915121680160789917 38 \times \cos(8n\pi/N) - 0.004307708101012136695 12442 \times \cos(10n\pi/N) + 0.000390411354137249556 8636 \times \cos(12n\pi/N) - 0.000015086135050228218 80403 \times \cos(14n\pi/N) + 0.000000132002427120203832 1705 \times \cos(16n\pi/N)$$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-233	-18	2.83	3.98	0.37	0.24	3.00



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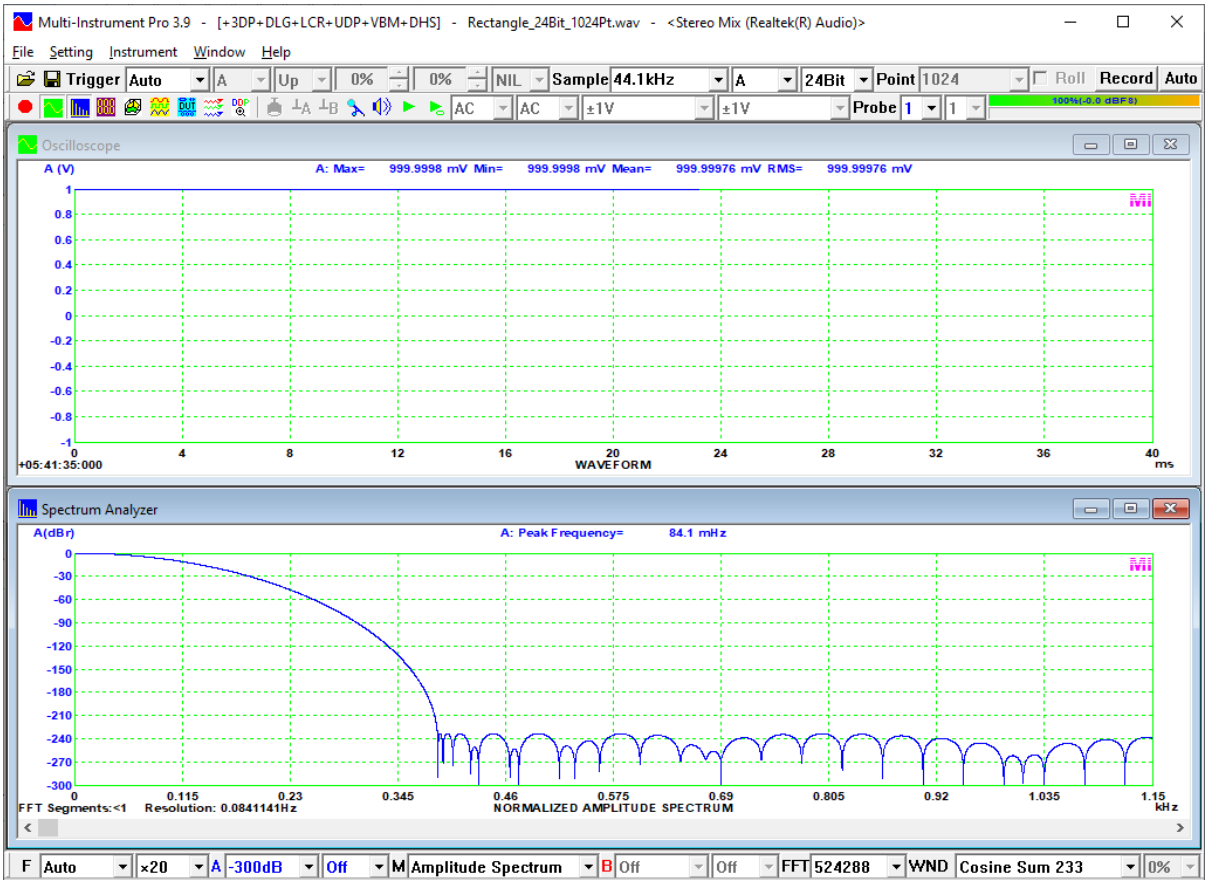

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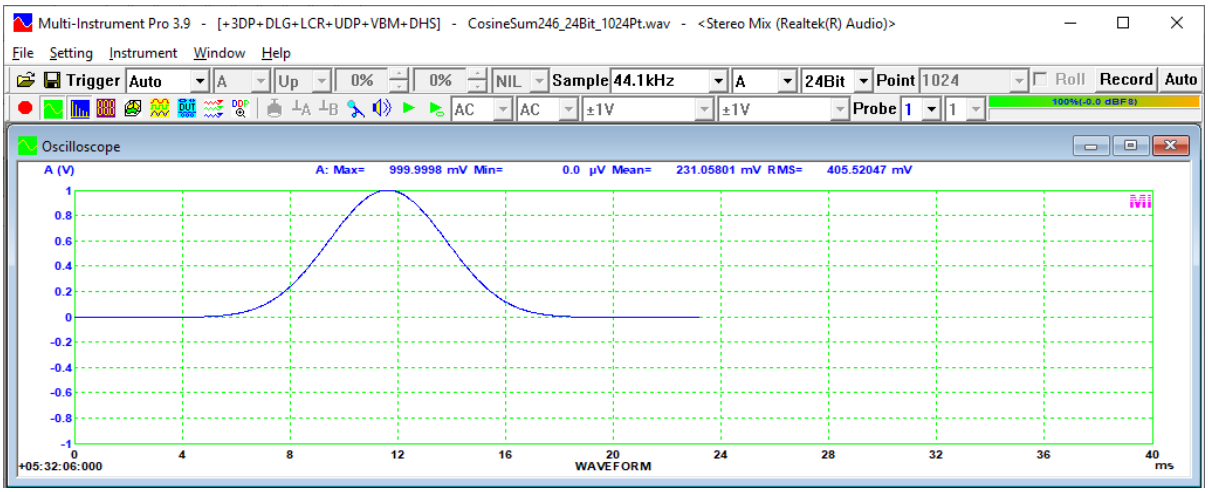
3.59 Cosine Sum 246 Window

$$w(n) = 0.231058120233135849943 5 - 0.392251473602165685883 1 \times \cos(2n\pi/N) + 0.238555362915897865559 7 \times \cos(4n\pi/N) - 0.102128866914911770697 9 \times \cos(6n\pi/N) + 0.029772941692923941858 33 \times \cos(8n\pi/N) - 0.005586700597441296634013 \times \cos(10n\pi/N) + 0.000612985169084468601634 3 \times \cos(12n\pi/N) - 0.000032957931642204056825 37 \times \cos(14n\pi/N) + 0.000000589988957874009604 2846 \times \cos(16n\pi/N) - 0.000000000953839042723873894 1684 \times \cos(18n\pi/N)$$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-246	-15	2.90	4.10	0.35	0.23	3.08



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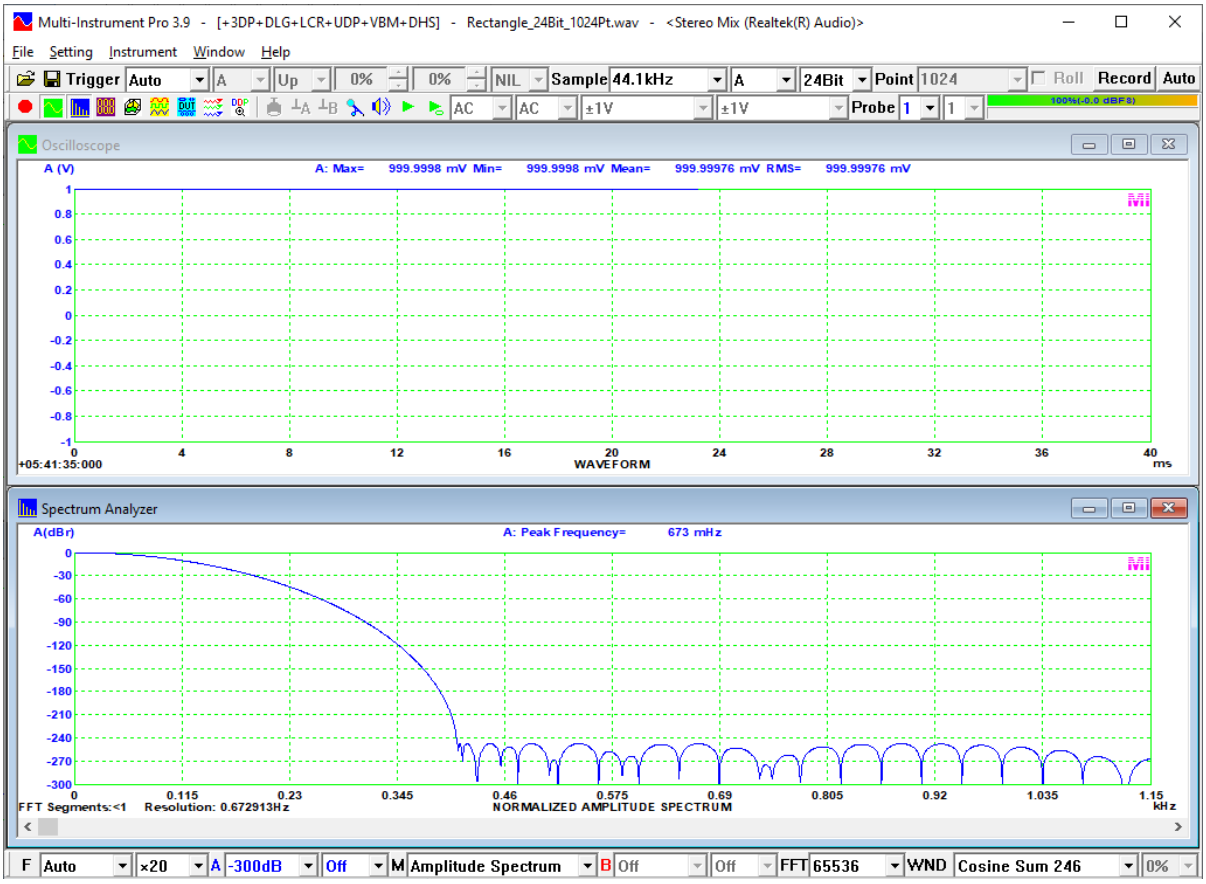

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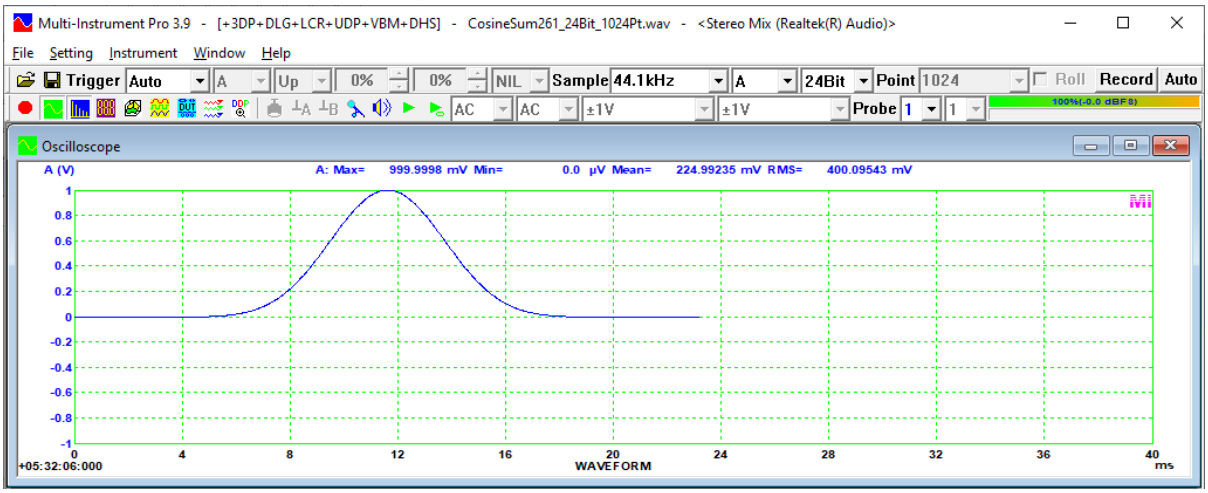
3.60 Cosine Sum 261 Window

$$w(n) = 0.224992461708753517732 9 - 0.385149542829290269325 9 \times \cos(2n\pi/N) + 0.240359768686502839096 8 \times \cos(4n\pi/N) - 0.107740807783785145478 1 \times \cos(6n\pi/N) + 0.033736306658002766212 90 \times \cos(8n\pi/N) - 0.007046059650969717333158 \times \cos(10n\pi/N) + 0.000909609134964287380448 2 \times \cos(12n\pi/N) - 0.000063578207637451814792 03 \times \cos(14n\pi/N) + 0.000001853811776589548714026 \times \cos(16n\pi/N) - 0.000000011528317416035633233 68 \times \cos(18n\pi/N)$$

$n = 0,1,\ldots,N-1;$

Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
-261	-12	2.98	4.20	0.34	0.22	3.16



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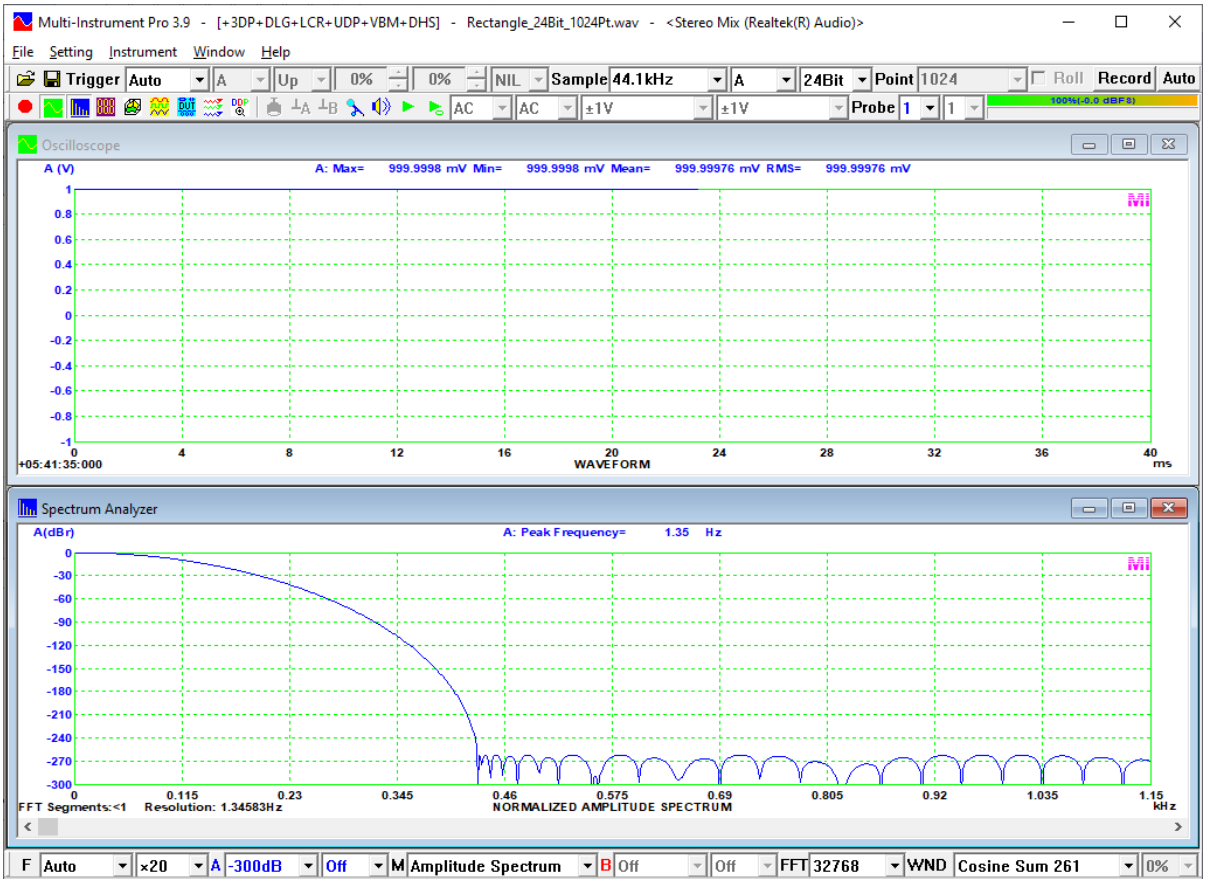

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4. Summary of Parameters of Window Functions

Window Function	Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
Rectangle	-13	-6	0.88	1.21	3.92	1	1
Triangle	-27	-12	1.28	1.78	1.82	0.5	1.33
Hanning	-32	-18	1.44	2.00	1.42	0.5	1.50
Hamming	-43	-6	1.30	1.81	1.75	0.54	1.36
Blackman	-58	-18	1.64	2.29	1.10	0.42	1.73
Exact Blackman	-68	-6	1.60	2.25	1.15	0.43	1.69
Blackman-Harris (4 terms)	-92	-6	1.90	2.66	0.83	0.36	2.00
Blackman-Nuttall	-98	-6	1.87	2.63	0.85	0.36	1.98
Flat top	-94	-6	3.72	4.58	0.012	0.22	3.77
Exponential ($\alpha = 0.1$)	-10	-6	1.03	1.47	2.83	0.39	1.41
Gaussian ($\alpha = 2.5$)	-43	-6	1.37	1.92	1.58	0.50	1.45
Gaussian ($\alpha = 3.0$)	-56	-6	1.60	2.26	1.16	0.42	1.71
Gaussian ($\alpha = 3.5$)	-71	-6	1.85	2.62	0.87	0.36	1.98
Welch (Riesz)	-21	-12	1.15	1.59	2.22	0.67	1.20
Cosine ($\alpha = 1$)	-23	-12	1.19	1.64	2.10	0.64	1.23
Cosine ($\alpha = 3$)	-39	-24	1.66	2.31	1.08	0.42	1.73
Cosine ($\alpha = 4$)	-47	-30	1.85	2.58	0.86	0.38	1.94
Cosine ($\alpha = 5$)	-54	-36	2.03	2.84	0.72	0.34	2.13
Riemann	-26	-12	1.25	1.73	1.89	0.59	1.30
Parzen (De La Valle-Poussin)	-53	-24	1.82	2.55	0.90	0.38	1.92
Tukey (Tapered Cosine) $\alpha = 0.25$	-14	-18	1.01	1.37	2.97	0.88	1.10
Tukey (Tapered Cosine) $\alpha = 0.50$	-15	-18	1.15	1.57	2.24	0.75	1.22
Tukey (Tapered Cosine) $\alpha = 0.75$	-19	-18	1.30	1.79	1.73	0.63	1.36
Bohman	-46	-24	1.70	2.37	1.02	0.41	1.79
Poisson ($\alpha = 2$)	-19	-6	1.21	1.70	2.03	0.43	1.31

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Window Function	Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
Poisson ($\alpha = 3$)	-25	-6	1.45	2.08	1.44	0.32	1.66
Poisson ($\alpha = 4$)	-31	-6	1.75	2.58	1.02	0.25	2.07
Hanning-Poisson ($\alpha = 0.5$)	-35	-12	1.53	2.14	1.26	0.43	1.61
Hanning-Poisson ($\alpha = 1.0$)	-39	-12	1.63	2.29	1.11	0.38	1.73
Hanning-Poisson ($\alpha = 2.0$)	N.A	-12	1.86	2.64	0.87	0.30	2.02
Cauchy ($\alpha = 3.0$)	-31	-6	1.34	1.91	1.67	0.42	1.49
Cauchy ($\alpha = 4.0$)	-36	-6	1.52	2.21	1.33	0.33	1.78
Cauchy ($\alpha = 5.0$)	-39	-6	1.69	2.54	1.11	0.27	2.07
Bartlett-Hann	-36	-12	1.40	1.94	1.52	0.50	1.46
Kaiser-Bessel ($\alpha = 0.5$)	-17	-6	0.95	1.31	3.32	0.85	1.02
Kaiser-Bessel ($\alpha = 1.0$)	-25	-6	1.11	1.53	2.43	0.67	1.15
Kaiser-Bessel ($\alpha = 2.0$)	-46	-6	1.43	1.99	1.45	0.49	1.50
Kaiser-Bessel ($\alpha = 3.0$)	-70	-6	1.70	2.39	1.02	0.40	1.80
Kaiser-Bessel ($\alpha = 4.0$)	-94	-6	1.94	2.73	0.79	0.35	2.05
Kaiser-Bessel ($\alpha = 5.0$)	-120	-6	2.16	3.03	0.64	0.31	2.28
Kaiser-Bessel ($\alpha = 6.0$)	-145	-6	2.35	3.31	0.54	0.29	2.49
Kaiser-Bessel ($\alpha = 7.0$)	-171	-6	2.53	3.56	0.47	0.27	2.68
Kaiser-Bessel ($\alpha = 8.0$)	-198	-6	2.70	3.80	0.41	0.25	2.86
Kaiser-Bessel ($\alpha = 9.0$)	-224	-6	2.86	4.03	0.37	0.23	3.03
Kaiser-Bessel ($\alpha = 10$)	-250	-6	3.01	4.24	0.33	0.22	3.19

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Window Function	Highest Side Lobe Level (dB)	Side Lobe Fall Off Rate (dB/Octave)	-3dB Main Lobe Width (bins)	-6dB Main Lobe Width (bins)	Scallop Loss (dB)	Coherent Gain	Equivalent Noise Bandwidth (bins)
Kaiser-Bessel (α = 11)	-276	-6	3.15	4.44	0.30	0.21	3.35
Kaiser-Bessel (α = 12)	-306	-6	3.29	4.64	0.28	0.20	3.49
Kaiser-Bessel (α = 13)	<-325	-6	3.42	4.82	0.26	0.20	3.63
Kaiser-Bessel (α = 14)	<-325	-6	3.54	5.0	0.24	0.19	3.77
Kaiser-Bessel (α = 15)	<-325	-6	3.66	5.17	0.22	0.18	3.90
Kaiser-Bessel (α = 16)	<-325	-6	3.79	5.34	0.21	0.18	4.03
Kaiser-Bessel (α = 17)	<-325	-6	3.90	5.50	0.20	0.17	4.15
Kaiser-Bessel (α = 18)	<-325	-6	4.01	5.66	0.19	0.17	4.27
Kaiser-Bessel (α = 19)	<-325	-6	4.12	5.81	0.18	0.16	4.38
Kaiser-Bessel (α = 20)	<-325	-6	4.23	5.96	0.17	0.16	4.49
Blackman-Harris (7 terms)	-180	<-6	2.48	3.50	0.48	0.27	2.63
Cosine Sum 220	-220	-36	2.90	4.08	0.36	0.23	3.07
Cosine Sum 233	-233	-18	2.83	3.98	0.37	0.24	3.00
Cosine Sum 246	-246	-15	2.90	4.10	0.35	0.23	3.08
Cosine Sum 261	-261	-12	2.98	4.20	0.34	0.22	3.16

The following steps can be used to compare the characteristics of different window functions graphically (see figure below):

1. Use “File Open” to open the WAV file of Rectangle window.
2. Use “File Combine” to open the WAV file of another window.
3. Set the settings for the Spectrum Analyzer properly as mentioned before
4. Push both curves in the Oscilloscope and Spectrum Analyzer to reference
5. Use “File Combine” to import another two WAV files
6. Push both curves in the Oscilloscope and Spectrum Analyzer to reference

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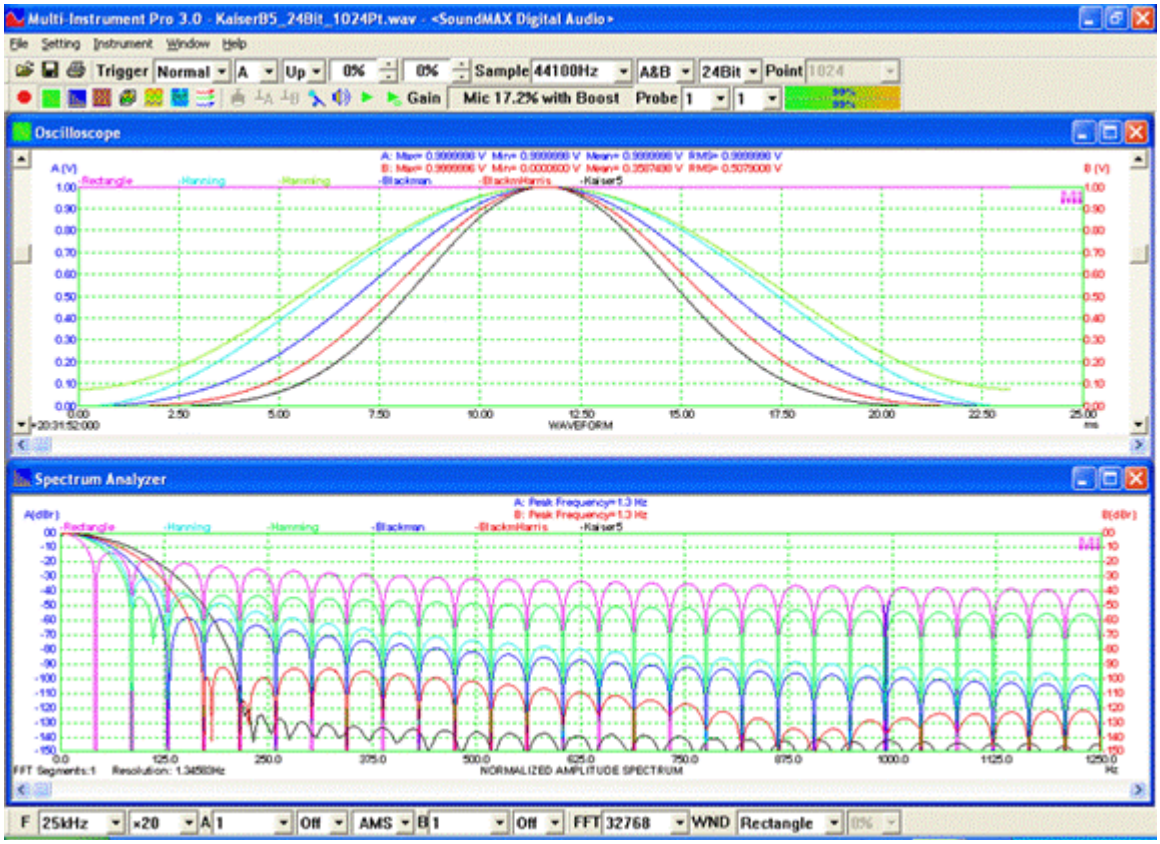
7. Repeat 5.



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