Insight into the gain-bandwidth product of amplifiers

By K Hayatleh, B L Hart and F J Lidgey

The gain-bandwidth product (GBP) of a conventional op-amp is a constant, or so we are led to believe. But does this mean that an inverting op-amp setup to attenuate with a voltage gain of 1/10 (-20dB) will operate successfully up to a frequency of ten times the op-amp's GBP? This seems unlikely and, indeed, it is not true.

Another puzzle is why is it that the bandwidth of an op-amp unity-gain inverter stage is only one-half that of the same op-amp connected to operate as a unity-gain non-inverting amplifier, i.e. a voltage-follower?

To answer these conundrums we must delve into the subtleties of the GBP. In seeking to obtain insight into GBP, we will need to analyse some amplifier circuits but we do so without resorting to advanced mathematics. With this objective in mind, an analysis employing phasor diagrams is used. These show, at a glance, the relative phases and magnitudes of signal currents and voltages in a circuit. All that needs to be recalled is that for sinusoidal signals the magnitude of the impedance of a capacitor, C, at a radian frequency, $\omega = 2\pi f$, is $1/\omega C$ and that capacitor current leads capacitor voltage by a phase angle of 90°.

Following a review of reasons for the existence of GBP constancy with a single-stage amplifier, there is a discussion of its relevance in voltage-feedback op-amp configurations and an explanation of the non-applicability of GBP constancy to current-feedback op-amps.

Single-stage voltage amplifier

Consider the cascode amplifier of **Figure 1a**, in which V_G is a sinusoidal input signal of angular frequency and rms amplitude V_G. Vo is the output signal. The voltage gain of T1 is close to unity and the only high impedance node is at the collector of T2. A simple small-signal model the amplifier can now be drawn (see **Figure 1b**). In this equivalent circuit $g_m (= 40mS/mA)$, the trans-conductance of T1 is proportional to the d.c. collector current; $\alpha (\approx 1)$ is the current gain of T2; C is the sum of the collector-base capacitance, C_{bc}, and the collector-earth stray capacitance, Cs, of T2. A phasor representation of the currents I_R, I_C in R and, C respectively, is shown in **Figure 2a**. $I_R = V_o/R$, represented by line OP, is in phase with V_0, which is taken here as the reference phasor. Since the current in C leads V_o by 90°, I_C is shown by the line PQ drawn perpendicular to I_R, and of magnitude of $Vo/_{(1/\omega C)}$, i.e $\omega C V_0$.

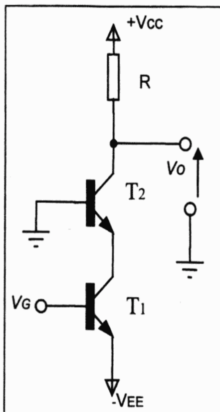


Figure 1a: A cascode amplifier and **Figure 1b (below)** its small signal equivalent circuit

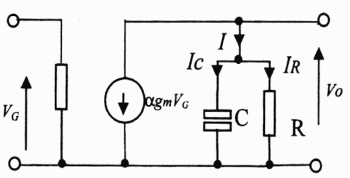


Figure 2a: A phasor plot for Figure 1b

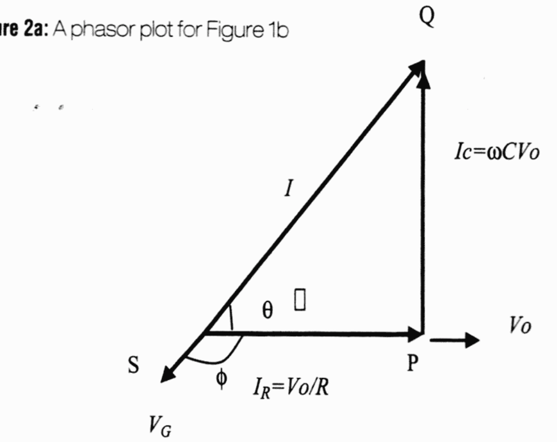
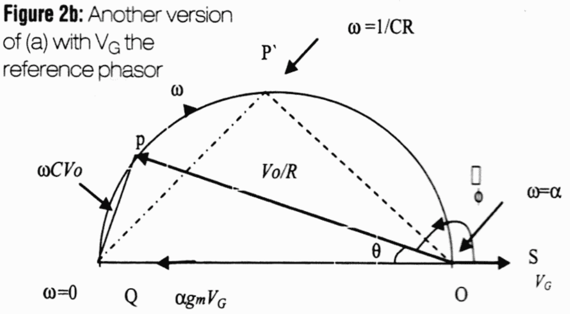


Figure 2b: Another version of (a) with V_G the reference phasor



The rule for the combination of phasors is the same as that in mechanics for the combination of differently directed forces. The line OQ represents $I = -g_m \alpha V_G$: as g_m and α are positive quantities the sign indicates for V_G a phasor OS oppositely directed to I. Using Pythagoras's theorem,

$$\left(\frac{Vo}{R}\right)^2 + (\omega C Vo)^2 = (\alpha g_m V_G)^2 \quad (1)$$

and, hence, the small-signal voltage gain, $G(\omega)$ is given by

$$|G(\omega)| = \left|\frac{Vo}{V_G}\right| = \frac{\alpha g_m R}{\sqrt{1 + (\omega CR)^2}} \quad (2)$$

Also, Vo leads V_G by ϕ, since phase is reckoned in the anti-clockwise direction.

Rotating Figure 2a in an anti-clockwise sense, so that V_G is our new reference phasor, gives **Figure 2b**.

If V_G remains constant as ω varies then OQ is fixed, in length, but OPQ is a right angle, hence the locus of point P as ω varies

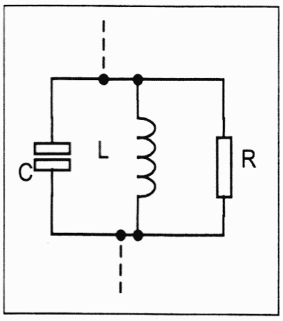
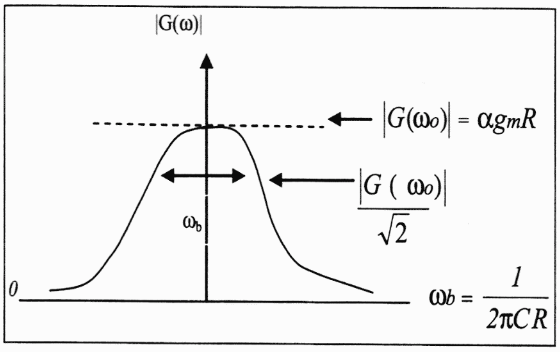


Figure 3a (Left): A tuned-circuit load, instead of R by itself

Figure 3b (Below): Showing the frequency-dependent voltage gain for (a)



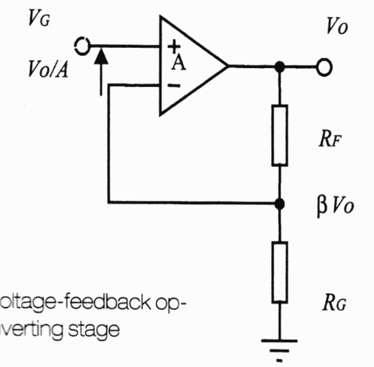


Figure 4: A voltage-feedback op-amp non-inverting stage

from O to ∞ is given by a semicircle constructed on diameter *OQ*.

From Equation 2, for $\omega \rightarrow 0$,

$$|G(o)| = \alpha g_m R \qquad (3)$$

Also, from Equation 2, $|G(\omega)|$ falls to $|G(0)|/\sqrt{2}$ for $\omega = \omega b$, where $\omega b CR = 1$ corresponding to point P\`
or,

$$fb = \frac{1}{2\pi RC} \qquad (4)$$

By definition, the cut-off frequency defines the -3dB bandwidth for voltage gain.
At fb, $\theta = 45°$, so $\phi = 135°$
Combining Equations 3 and 4,

$$GBP = |G(o)| \times fb = \frac{\alpha g_m}{2\pi C} \qquad (5)$$

This means that GBP is constant because $|G(0)|$ is proportional to R and the frequency function determining fb has R in its denominator, and so GBP is independent of R.

Historical note

Before the advent of transistors, the figure of merit g_m/C was often used as a criterion for the selection of pentode valves in video amplifier design. In that case, C referred to inter-electrode capacitance and g_m the mutual conductance of the valve.

With respect to bipolar transistors, all types working at a specified temperature and collector current, in the low milliampere range, have effectively the same g_m, but C_{bc} is smaller for high frequency devices, which have smaller junction areas.

In this discussion, so far, the frequency response of the transistors has been ignored in the calculation of f_b. This is justified because the characteristic, or transition frequency, which even for run-of-the-mill devices is likely to be 500MHz or above. For a discrete device design in which R=5kΩ and C=2pF Equation 5 gives $f_b \approx$ 16MHz which is negligible compared to 500MHz.

In circuit theory terms, the load circuit produces a dominant pole, corresponding to $f_b \approx$16MHz, in the system response.

In concluding this review of the single-stage amplifier, it is noteworthy that the constancy of GBP also applies to the simple tuned amplifier which results when R is replaced by the parallel RLC circuit of **Figure 3a**; the response for this is shown in **Figure 3b.**

A voltage op-amp non-inverting stage

Consider the op-amp non-inverting stage of **Figure 4**. Subject to the usual op-amp assumptions we can write, by inspection,

$$Vo\left[\beta + \left(\frac{1}{A}\right)\right] = V_G \qquad (6a)$$

Where $\beta = R_G/(R_F+R_G)$ is the feedback factor and A is the open-loop gain of the op-amp.

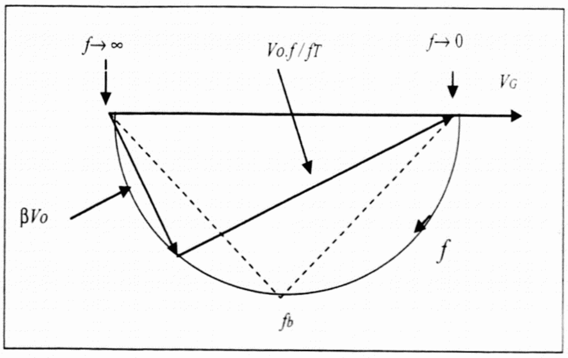
Equation 6a can be rearranged to give an expression for the voltage gain G that is used in later discussion, or

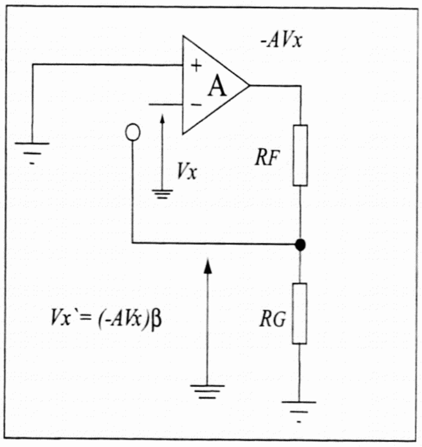
$$G = \left(\frac{Vo}{V_G}\right) = \left(\frac{\frac{1}{\beta}}{\left[1 + \left(\frac{1}{A\beta}\right)\right]}\right) \qquad (6b)$$

For low frequencies well below the cut-off frequency, fo, of the open-loop op-amp, that is $f << fo$, then A is given by $A = A0 >> 1$, so $|G(0)| \cong (1/\beta)$

$$A = A0 >> 1, \text{ so } |G(o)| \cong \left(\frac{1}{\beta}\right)$$

From the data sheet characteristics of commonly used op-amps it is evident that, for $f >> fo$,





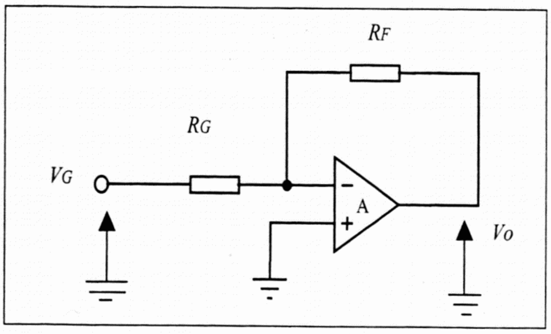


Figure 5 (Top): Phasor diagram for Figure 4 (for AO>>1/β)
Figure 6 (Middle): The loop gain for Figure 4 is $\left(\frac{V_x'}{V_x}\right)$
Figure 7 (Bottom): A voltage-feedback inverting amplifier

$$A = \left(\frac{A_0 f_0}{f}\right)\angle -90^0 \qquad (7)$$

$|A|$ =1 when $f=f_T$, say, so $f_T=A_0 f_0 \approx 1MHz$ for the 741 op-amp
Hence,

$$\frac{V_o}{A} = V_o \frac{f}{f_T}\angle 90^0 \qquad (8)$$

A phasor plot for Equation 6a using Equation 8, constructed in a similar manner to that in Figure 2b, is shown in **Figure 5**. The cut-off frequency f_b, as previously, occurs at the mid-point of the semicircle, so $\beta V_0 = V_{0fb}/f_T$. (Note that the semicircle is now below the horizontal axis because we are dealing with a non-inverting amplifier).

Consequently,

$$f_b * \left(\frac{1}{\beta}\right) = f_b * |G(o)| = f_T = \text{constant} \qquad (9)$$

The GBP is constant for a reason similar to that for a simple amplifier. In this case, $|G(0)|$ is dependent on a resistor ratio and the same resistor-ratio occurs in the denominator of the frequency function determining f_b.

This can also be seen by examination of Equation 6b: the frequency variation of the closed loop gain is dependent on $1/A\beta$. Now $-A\beta$ is the voltage loop-gain for the circuit as in **Figure 4**. It is a property of the loop alone and not related to the input signal, so it applies to the inverting amplifier configuration of **Figure 7** as well as Figure 4. It is determined by cutting the loop at some convenient point, inserting a test signal, V_x', and finding the signal, V_x', occurring at the other side of the cut: loop gain = V_x'/V_x (see **Figure 6**). Thus, for the non-inverting op-amp configuration, the GBP is constant because the loop-gain is proportional to β and the closed-loop gain is inversely proportional to β. At $f=f_b$ the magnitude of the voltage loop-gain is unity.

A voltage op-amp inverting amplifier

For the inverting amplifier stage in Figure 7, the well-known and easily derived expression for $G=(V_o/V_G)$ is

$$G = -\frac{\left(\frac{R_F}{R_G}\right)}{\left[1+\left(\frac{1}{A\beta}\right)\right]} \qquad (10)$$

It is apparent that the frequency response is governed by the same factor $1/A\beta$ as in the case of the non-inverting stage. However, there is a difference because the low frequency closed-loop gain $|G(0)|$ is equal to R_F/R_G, not [assuming as before, that $A_o >> 1$]
But,

$$|G(o)| = \left[\frac{R_F+R_G}{R_G}\right] - 1 = \left[\left(\frac{1}{\beta}\right) - 1\right] \qquad (11)$$

hence

$$\left(\frac{1}{\beta}\right) = \left[|G(o)|+1\right] \qquad (12)$$

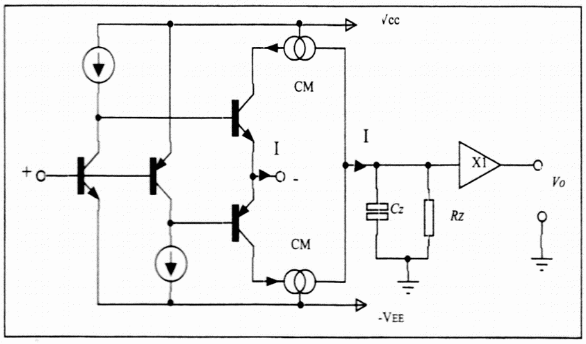
As before, from equation (9), $f_b(1/\beta)=f_T$
Substituting for $(1/\beta)$ from Equation 12,

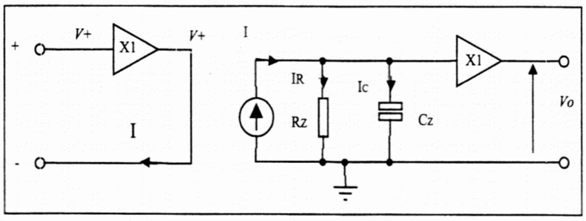
$$f_b * \left[|G(o)|+1\right] = f_T \qquad (13)$$

For large values of $|G(0)|$ there is a little difference between this and Equation 9. However, for low values of $|G(0)|$ there is a significant difference.

In the case of a non-inverter, strapped as a voltage-follower, $f_b=f_T$, however, for a unity-gain inverter. What is the physical explanation for this?

The answer to this question, which was posed at the beginning





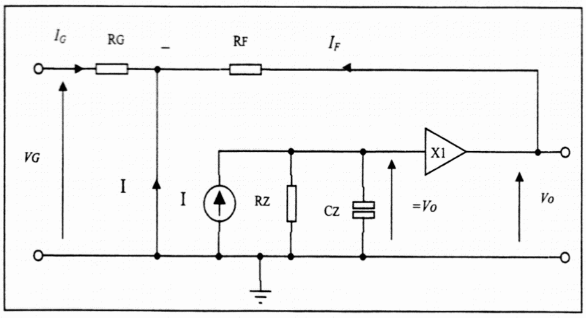


Figure 8 (Top): Basic schematic of a CFOA

Figure 9 (Middle): Small signal equivalent circuit for Figure 8

Figure 10 (Bottom): Equivalent circuit of a CFOA, based on Figure 9

of this article, is that for the voltage follower, $\beta=1$, i.e there is a maximum feedback, but for the unity-gain inverter $\beta=0.5$, i.e. less feedback and, consequently, less closed-loop bandwidth.

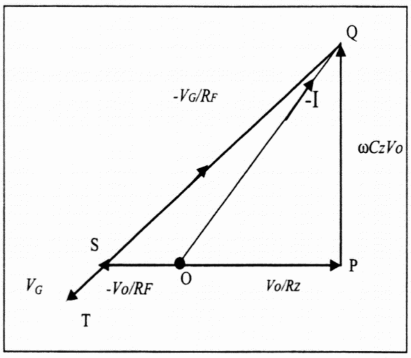
Current-feed back op-amp configurations

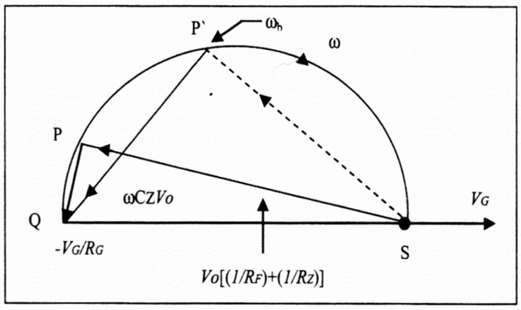
A schematic of a simple current-feedback op-amp (CFOA) is shown in **Figure 8**: R_Z, C_Z are not necessarily added components but represent the effective resistance and capacitance seen at the input of the output voltage-follower stage. A simple small-signal equivalent circuit for the inverting configuration is shown in **Figure 10**. The analysis is similar to that presented earlier.

Referring to **Figure 11a** (not shown to scale), OP represents the current in R_Z and PQ the current in C_Z. These sum to $-I$ represented by OQ. However, by inspection of Figure 10, the error-current I is given by

$$I = -I_G - I_F = -\left(\frac{V_G}{R_G}\right) - \left(\frac{V_O}{R_F}\right) \quad (14)$$

In **Figure 11a**, OS= $-(V_O/R_F)$ so $-(V_G/R_F)$ must be SQ is in order to satisfy Equation 14: ST, oppositely directed to SQ, must therefore represent the direction of V_G. Rotating the diagram, anti-clockwise, as in the case of Figure 2b, gives **Figure 11b**.





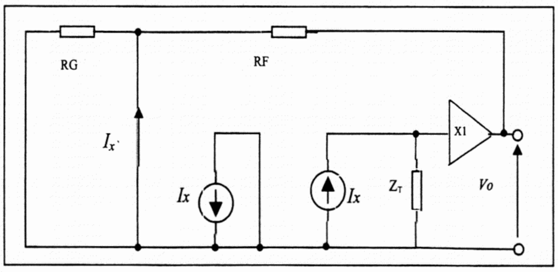


Figure 11a (Top): Phasor plot for Figure 10

Figure 11b (Middle): Another version of (a) with VG as the reference phasor

Figure 12: The loop-gain for the CFOA is $\left(\frac{I_X'}{I_X}\right) = -\left(\frac{Z_T}{R_F}\right)$ for test signal Ix (ZT represents RZ in parallel with CZ)

Again, as ω varies, the locus of P is a semicircle on the diameter QS. The -3dB bandwidth frequency, f_b, occurs at P` where $\omega_b(=2\pi f_b)$is given by

$$\omega_b C_Z V_O = V_O\left[\left(\frac{1}{R_F}\right)+\left(\frac{1}{R_Z}\right)\right] \quad (15)$$

or,

$$f_b = \frac{1}{2\pi C_Z R_Q} \quad (16)$$

where, $R_Q=R_F/R_Z$ so,

For the reason discussed below, $R_Z>>R_F$, normally, so

$$f_b \approx \frac{1}{2\pi C_Z R_F}$$

This means the current in R_F is equal in magnitude to the current in C_Z at $f = f_b$, or the magnitude of the current loop-gain is unity. This is independent of R_G, so the product of the low frequency closed loop gain and f_b is not constant. The gain can be varied by changing R_G , but this does not effect f_b. This is true for the non-inverting configuration, as well.

Glancing again at Figure 11b, it is apparent that altering R_G changes the diameter of the semicircle but does not alter the relative position of P` on its perimeter.

Another way of appreciating the non-constancy of GBP is to look at the general expression for gain. If in Figure 10 we replace R_Z and C_Z by an equivalent impedance Z_T, then we can rewrite Equation 14 as,

$$I = \left(\frac{Vo}{Z_F}\right) = -\left(\frac{V_G}{R_G}\right) - \left(\frac{Vo}{R_F}\right)$$

Rearranging this gives,

$$G = \frac{Vo}{V_G} = -\frac{\left(\frac{R_F}{R_G}\right)}{\left[1+\left(\frac{R_F}{Z_T}\right)\right]} \tag{17}$$

In this, $-(Z_T/R_F)$ is the current loop-gain (see **Figure 12**), so Equation 17 is similar in form to Equation 10. However the frequency-dependency of the low frequency loop-gain magnitude (R_Z/R_F) does not involve R_G. We normally require that (R_Z/R_F) be large for good definition of low frequency closed-loop gain.

However, a limitation on the magnitude of R_Z, normally specified by the op-amp manufacturer, is required for Nyquist stability. It has to be remembered that Figure 9 is a simplified schematic and that in addition to the pole introduced by R_Z and C_Z there are additional CFOA poles due to the frequency response of the constituent current-mirrors and voltage-followers of the CFOA.

The constant GBP limitation for the simple voltage amplifier and voltage-feedback op-amp configurations is circumvented with current-feedback op-amps, because the factor governing low frequency loop-gain does not appear in the frequency-dependent expression that determines the bandwidth.

Conclusions

The above analysis shows how the GBP for a conventional transistor voltage amplifier, whether it is a single stage transistor amplifier or an op-amp, is constant. Also, it has been shown that the feedback factor is the same whether an op-amp is used in an inverting or non-inverting configuration. It is left to the reader to work out the expected bandwidth of the -20dB inverting attenuator. Finally, the current-feedback op-amp analysis shows that in this architecture of op-amp, the constant GBP rule of the voltage amplifier can be broken, with the gain and bandwidth separately controllable.